Modelling of Stratified River Bank Erosion Due to Undercutting

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Abstract: Theories of soil mechanics and basic hydraulic relations and principles of loose boundary channel hydraulics were combined and integrated to analyze stratified river bank failure due to undercutting (undermining) of a bottom loose layer. A near bank velocity distribution model was developed from which the boundary shear stress acting on the bank surface was determined. The model allows computation of the eroded soil volume from the bottom loose layer and the lateral undercut distance at any time in the flow hydrograph as long as the induced flow shear stress is greater than the particle critical shear stress resistance to hydraulic entrainment. At failure the size of the failure block and the ultimate critical value of the lateral undercut distance are determined. Therefore the annual rate of bank retreat and bank sediment load contribution can be identified for the reach under consideration. The application of the model to the River Nile hydrograph in Northern Sudan State showed a good agreement with field observations.

Keywords: stratified bank; driving force; resisting force; failure plan; undercutting; near bank velocity distribution; induced boundary shear stress.

1. INTRODUCTION

River bank erosion processes have negative socioeconomic impacts on the people living along and adjacent to river banks. Riverbank erosion results in land loss, damage to people properties, loss of fruit gardens, dates and continuous threats to buildings, bridges, and pumps intakes. In northern Sudan many families were forced to abandon their lands due to bank erosion. If the fertile lands form small strip extending along the bank of the Nile then one can imagine how severe is the impact of land loss on the families living along the Nile banks.

Three processes of bank erosion can be identified in general. When the channel bed is not subject to erosion but its cohesive banks erode then we will have lateral erosion only. When the channel bed in the vicinity of the banks is eroded (degradation or local scour) but the banks do not erode then we will have a case of bank heightening which later results in a mass failure of the banks. When the processes of bank heightening and bank lateral erosion are combined then the rate at which the bank is eroded and failed is highly accelerated (typical to the case that occurs at outer alluvium channel bend).

The River Nile in Sudan and two of its tributaries; namely, the Blue Nile and River Atbara have cohesive banks resting on sandy river bed. The cohesive banks are stratified and their colors vary from dark brown to black color. The Nile banks subject to severe banks erosion attributed mainly to the presence of a weak or loose stratum underlying the top soil. At river bends where the induced flow shear is high this layer is eroded causing failure of the top soil [1, 2]. Field and laboratory investigations revealed that for many locations subjected to bank undercutting the bottom layer consists of very fine sand with apparent cohesion [2]. The apparent cohesion is due to the presence of some silt, clay and consolidation resulting from the load of the top layers. The apparent cohesion diminishes when this layer becomes wet.

The process of the Nile bank erosion is a seasonal one occurring only during flood season (July-September). The failure process goes through three distinct time stages (Based on the erosion rate) which take place in the period from July to October, Fig. 1:

1- High at the beginning of the flood rising (period of 3 - 4 weeks).
2- Almost ceases at high flood levels (period of 4 -5 weeks).
3- It resumes as the flood recedes but at a very low rate compared to stage of flood rising.

The erosion process ceases at the low flow period (November - June). However, sometimes post failures occur but are rare.

The high erosion rate at the beginning of the flood is attributed to the increasingly induced flow shear at the outer bend where thawleg line directly impinges on the banks. At this stage the bottom layer is scoured producing an overhanging block (cantilever) which soon fails as shown in this paper. The fallen
block rests at the bank toe and helps preventing further under cutting of the bank. By the time the flood reaches its maximum levels the thaw line shifts towards the central channel and the undercutting process ceases. The majority of the bank failures occurred at the recession stage is due to increased bulk weight (saturated soil), pore pressure, and seepage forces [3, 2].

Another important feature of cohesive banks of the Nile in Northern Sudan is the development of tension cracks. Presence of tension cracks weakens the bank resistance to erosion and negatively affects the strength of the soil especially when the cracks are filled with water.

In this paper failure of stratified cohesive banks due to undercutting is analyzed. The study is limited to the first stage of flood rising mentioned above. Accordingly, the analysis is carried out assuming dry banks. The induced hydraulic forces, hydrostatic pressure force, buoyant force, gravity forces and bank soil resistance are combined together to develop a model that simulates stratified bank failure due to undercutting. The main objective of the model for the different failure cases considered below is to compute the lateral undercutting distance \( \Delta L \) at which the bank becomes unstable. This leads to the determination of the volume of the failure block, sediment amount (contributed by the banks), and lateral eroded distance (bank retreat).

2. THEORY

Numerous works have been carried out in the area of river bank erosion [4-15]. Engineers, morphologists, geologists and others made great effort to answer the questions: what causes a river bank to erode? Answering the question contributes to answering the question of why rivers meander and what are the suitable measures to protect the banks against erosion.

The factors that affect the stability of the river banks and cause erosion are well known. Such factors are: steep bank slope, presence of tension cracks, wind waves, boats waves, large bank height, bank heightening due to bed lowering, local scour, large induced flow shear, pore pressure, increased bulk weight of soil, surcharge loads, type of bank soil and stratification, location at river bends, and water level fluctuation.

Water and soil are integrated together to analyze the stability of river banks. It is not only the chemical and physical properties of the soil that need to be considered in the analysis but also the mechanics of failure and the hydraulic characteristics of the flow.

Slope Stability: the Factor of Safety:

In relation to channel bank stability analysis the factor of safety FS for plane failure surface shown in Fig. 2 below is defined by:

\[
FS = \frac{F_R}{F_D} = \frac{\text{resisting force}}{\text{driving force}}
\]

Or

\[
F_D = \frac{F_R}{FS} = \frac{C' \times LL + N \times \tan\phi'}{FS} = \frac{C' \times LL + N \times \tan\phi'}{FS} = \frac{C \times LL + N \times \tan\phi}{FS}
\]

Where:

\[
C = \frac{C'}{FS} \quad \text{and} \quad \tan\phi = \frac{\tan\phi'}{FS}
\]

![Fig.1. Nile hydrograph at Merowe (1912 – 2000)](image1)

![Fig.2a. Undercut bank geometry at failure](image2a)

![Fig.2b. Driving and resisting forces along failure plane EF](image2b)
LL= length of the failure plane EF (later called EO)  
N= Force component normal to the failure plane EF  
C= Effective soil cohesion  
Φ = effective internal angle of friction

The resisting force $F_R$ is attributed to effective soil cohesion $C$ and internal angle of friction $\Phi$. However, when the water level is high enough the buoyant and hydrostatic forces components (from the river side) contribute to the failure resistance as explained below.

$F_B$ is the driving force that tends to draw the soil block downward (failure). It consists of weight component of failure block and hydrostatic force components resulting from the tension crack when it is filled with water in addition to any surcharge loads posed on the banks (building, trees ...etc). If the water level remains higher for a long period the soil becomes saturated and upon flood recession the pore pressure and saturated bulk weight of the soil contribute significantly to the bank instability. It worth to mention the fact that many rivers get their sections run fully or with over bank flow during the flood season which last for a few months as in the case of the Nile (3 months flood duration). The rest of the year the water level is very low and the banks become dry (8 months low flow duration). At the beginning of flood bank failure takes place. In such situation the soil properties are regarded as for dry bank with zero pore pressure.

**Stratified Banks:**

Fig. 3 below depicts the case of a stratified river bank consisting of n layers above the river bed. The $n^{th}$ layer is easily eroded layer (usual fine to medium sand with apparent cohesion). The layers above the $n^{th}$ layer are classified as cohesive and highly resistant to erosion [4]. For each layer the thickness $L$ and the soil properties such as the effective cohesion $C$ and specific weight $\gamma$ are known. The bank height is $H_o$ with inclination angle $i$. The top of the $n^{th}$ layer is located at a depth $h_1$ below the river water surface and at vertical distance $L_o$ below the top of the bank. In other words, the thickness of the $n^{th}$ layer $L_n$ is equal to $H_o - L_o$.

Provision of weighing averaging of properties of cohesive top layers is recommended when possible for ease of computations. When such a situation is maintained one can deal with the bank as if it consists of a single cohesive layer on the top of the erodible layer.

**Fig.3. Initial bank geometry**

**Fig. 4.** Hereafter shows the condition at failure due to lateral undercutting at the bottom layer.

**Fig.4. Working forces near failure**

At or near failure the working forces are per unit channel width:

$W =$ self weight of the failure block in kN/m  
$F_B =$ buoyant force in /m  
$F_{h1} =$ hydrostatic force river side in kN/m  
$F_{h2} =$ hydrostatic force due to water in tension crack in kN/m

The block failure (failure mode) occurs either by having the fallen block slides along a developed failure plane as shown in Fig. 4 or just overturns at point O giving what is known as moment failure.

In the following paragraphs the methods of analysis of the two mentioned modes of failure are carried out. The objective is to determine the lateral undercut distance $\Delta L_o$ at which mass failure of the bank takes place.

**I- Failure block slides on the failure plane:**

At failure, $FS=1$. Then, along the failure plane EO (values are per unit channel length), Eq. [1] gives:

$$F_B = F_H$$

Substituting for the force components using Fig (4), then

$$(W - F_B) \sin \beta - (F_{h1} - F_{h2}) \cos \beta = [(C_1 \cdot L_1 + C_2 \cdot L_2 + \ldots C_{n-1} \cdot L_{n-1}) - y \cdot (C_1 + C_2 + \ldots C_m)] \cdot \cot \beta + NT \Phi \ldots [2]$$

Where

$$W = \sum_{j=1}^{n-1} w_j$$

$w_j$ is the weight of the $j^{th}$ stratum per unit channel length given as
\[ w_j = (A_j \ast 1) \ast y_j = l_j \ast \left( \frac{BW + \gamma_{w,i+1}^{\text{th}} l_j}{\tan i} \right) \ast y_j \quad \text{for} \quad 1 \leq j \leq m \]  \hfill [3.2]

\[ w_j = (A_j \ast 1) \ast y_j = l_j \ast \left( BW + \gamma y_{\tan i} + \left( \sum_{k=m+1}^{l} k \ast \frac{l_j}{2} \right) \ast \tan i \right) \tan \beta \ast y_f \quad \text{for} \quad m+1 \leq j \leq n-1 \]  \hfill [3.3]

\( y_i \) = weight density of material of the jth stratum.

\( m \) = refer to the mth stratum at which the tension crack depth ends at its bottom.

\[ F_{h1} = \gamma_{w} \ast h_1 \ast \left[ \Delta L \ast \tan i - L_n - \frac{h_1}{2} \right] / \tan i \]  \hfill [4.1]

\[ F_{h2} = \frac{1}{2} \gamma_{w} h_2^2 \]  \hfill [4.2]

\[ F_{h3} = \frac{1}{2} \gamma_{w} h_3^2 \]  \hfill [4.3]

\( \gamma_{w} \) = specific weight of water

\[ NT \Phi = N \ast \tan \Phi_a \]  \hfill [5.1]

\( \Phi_a = \) average value of \( \Phi \) or a preselected value

\[ N = W \cos \beta + (F_{h1} - F_{h2}) \ast \sin \beta = W \cos \beta + \frac{1}{2} \gamma (h_1^2 - h_2^2) \ast \sin \beta \]  \hfill [5.2]

In case where there is a significant variation in the values of \( \Phi \) then \( NT \Phi \) can be computed based on summing up the normal loads corresponding to the layers. In other words:

\[ NT \Phi = \sum_{j=1}^{n} N_j \ast \tan \Phi_j \]  \hfill [5.3]

\[ N_j = \left[ \sum_{k=1}^{j-1} \gamma_k \ast L_k + y_j \ast \frac{l_j}{2} \right] \ast L_j \ast \cot \beta \]  \hfill [5.4]

The volume of failure block BV in \( m^3/m \)

\[ BV = \sum_{j=1}^{l} \frac{w_j}{y_j} \]  \hfill [6]

Since \( L_o = H_o - L_n = \sum_{j=1}^{n-1} L_j \)

Then \( \Delta L = BW + \frac{H_o}{\tan i} - \frac{W - y}{\tan \beta} \) \hfill [7]

Eq. [7] provides at failure the geometric relation between the critical distance \( \Delta L \), the failure block width \( BW \), tension crack depth \( y \) and the angle \( \beta \) of the failure plane EO. Eq. [7] can be written in the form

\[ \beta = \tan^{-1} \left( \frac{W - y}{BW + \frac{H_o}{\tan i} - \Delta L} \right) \]  \hfill [8]

The solution of Eq. [8] for angle \( \beta \) allows computation of the values of \( BW \) and \( BV \) for the failure block. Consequently, the resulting geometry of the new bank slope is identified and the bank retreating rate and the sediment load contributed by the banks can be determined. The critical value \( \Delta L \) at which the bank fails is equal to the total of the incremental values of the under cutting distance \( \Delta L \) computed from the sediment model corresponding to a time step \( \Delta t \). In other words

\[ \Delta L = \sum_{j=1}^{k} \delta L_j \]  \hfill [9]

And the time to failure, \( t_f \) is

\[ t_f = \sum_{j=1}^{k} \Delta t_j \]  \hfill [10]

Where \( k \) is the required number of time steps to arrive at failure. For constant value of At the time to failure \( t_i \) is equal to \( k^*At \).

It should be noted that \( \delta L \) is a function of the flow characteristics such as the velocity, induced flow boundary shear, soil properties and critical shear resistance to erosion of the \( n \)th layer. It is natural to have the \( n \)th layer at the bottom consisting of fine sand as in the case of the Nile. Therefore the appropriate sediment transport relation for such a type of soil should be chosen when carrying out the analysis. The solution steps are demonstrated below.

When the failure plane is initiated at the bottom of a tension crack of depth \( y \) the fallen block width \( BW \) is automatically defined as shown in Fig. In this case, solution of Eq. [8] at failure \( (F_{h1} > F_{h2}) \) gives directly the failure plane angle \( \beta \) for computed values of the undercut distance \( \Delta L_e \). In other words, angle \( \beta \) is a function of the undercutting distance \( \Delta L \) while the erosion process is ongoing. However, if no tension cracks developed, then at failure the angle \( \beta \) is a function of both the undercut distance and \( BW \).

**II- The failure block overturned at point \( O \):**

In this case the forces and their magnitudes and directions are similar to those developed for the first failure mode. The pivotal point of the lever arms for the computation of the moments of the forces is at point \( O \) as shown in Fig (5). However, at the instant of failure the role of the resisting force \( F_R \) diminishes.

At critical state and for failure block overturning around point \( O \) then:

\[ F_{h1}*z_1 + F_{h2}*z_4 - W*z_3 - F_{h2}*z_2 = 0 \]  \hfill [11]

Where \( W \) is the weight of failure block per unit channel length and \( z_3 \) is the line of action of \( W \), or

\[ F_{h1}*z_1 + F_{h2}*z_4 - \sum w_j*z_3 - F_{h2}*z_2 = 0 \]  \hfill [12]

![Fig.5. Working forces and moments near failure](image-url)
Where \( w_j \) is the weight of the \( j^{th} \) stratum and \( z_{j3} \) is the normal distance of the line of action of \( w_j \) from point \( O \), Fig. 6. The expressions for the hydrostatic forces, the buoyant forces and the layers weights are as given before.

The bottom width \( \xi \) of the \( j^{th} \) stratum in the presence of tension cracks is given by

\[
\xi_j = BW + \sum_{j=m+1}^{n} L_j \left( \frac{1}{\tan i} - \frac{1}{\tan \beta} \right) + \frac{\sum_{k=m+1}^{n} L_k}{\tan i} \quad \text{for} \quad m + 1 \leq j \leq n - 1 \]  

[14]

For simplicity the lever arm for moment calculation of each stratum can be approximated as equal to \( \xi_j / 2 \).

If the tension cracks do not exist then set \( y=0 \) or \( m=0 \) in the above relation to get the width of any stratum.


**Tension crack depth:**

Soil mechanics literature informed that the depth of the tension crack \( y \) is a function of the cohesion \( C \) and specific weight \( \gamma_s \) of the soil. Certain relation exists such as the one below.

\[
y = \left[ \frac{2C}{\gamma_s} \right] \tan(45^\circ + \Theta / 2) \]  

[15]

Or simply

\[
y = \frac{2C}{\gamma_s} \]  

[16]

For computation of the tension crack depth \( y \) it is suggested for cohesive stratified river banks to adopt a representative cohesion value based on

\[
C = \frac{\Sigma_{j=m+1}^{n} L_j}{\Sigma_{j=1}^{n} L_j} \]  

[17]

However, some researchers limit the depth of tension crack not to exceed half of the bank height [5]. Determination and adjustment of the tension crack depth near failure depends largely on field observation and experience.

**Some common observed field cases:**

The two cases presented below are commonly observed to occur at site. However, they can be considered simplified cases of the above general case where \( \beta=90^\circ \) (i.e. vertical failure plane).

1- **No tension cracks developed**

This means \( y=0 \) and consequently \( h_2=0 \). Field observations reveal Fig. 7 below where \( \beta=90^\circ \).

Accordingly,

\[
L = BW + \frac{H_0}{\tan i} \]  

[18]

2- **Dry tension crack developed**

This means \( y>0 \) but \( h_2=0 \). Fig. 8 below depicts the case where \( \beta=90^\circ \).

Then

\[
\Delta L = BW + \frac{H_0}{\tan i} \]  

[19]
Boundary Shear stress and near bank velocity distribution:

For river bank located at the outer side of a bend, the flow shear stress acting on the bank is a result of the gravitational effect, secondary current and turbulent intensity [16]. To better estimate the flow shear stress acting on the bank slope and the adjacent river bed the velocity distribution in the zone influenced by the bank needs to be identified. This requires solution of hydrodynamic equations for curvilinear flow to obtain the velocity distribution over the entire height of the bank. Specifically, it is required to find the magnitude of the induced flow shear force on the part of the bank slope which is prone to particles removal (eroded layer). The analysis this part corresponds to n\textsuperscript{th} layer mentioned above. In other words, the required model should integrate flowing water and bank and bed soil material to arrive at reasonable answer to the question of how to quantify river bank undercutting in terms of cut distance \( \Delta L \) and contributed sediment amount.

Many research works have been carried out to determine the boundary shear stress distribution for straight and meandering channels. Other works investigates also determination of the near bank velocity distribution [13, 14, 16], Blanckaert and De Vriend [17] carried out an experimental study where they found out for flow in river bends besides the classical helical motion (centre-region cell), a weaker and smaller counter-rotating circulation cell called outer-bank cell is observed near the outer bank and believed to contribute to bank erosion process. They concluded that the profiles of the downstream velocity in the outer bank region are similar to those in the central region. Moreover, the exchange of energy between mean flow and turbulence is quite small in this region. Jason et al. [13] carried out flume study to investigate the near-bank velocity distribution for river banks with berms. This work mainly considers incorporation of the form drag and the hydrodynamic resistance of small scale topographic features commonly found on the banks with berms of natural channels. The results are useful for the analysis of staggered banks (stair type of banks). In the case of river banks subject to undercutting the geometry of the bank slope is different from that of bank with berm as in Jason study. Therefore, a different approach needs to be applied to determine the velocity and shear distribution near the bank. The ray- isovel approach developed by Kean and Smith [18] is aimed at finding the boundary shear stress and velocity distribution for a channel cross section. The wetted section is gridded into rays which start at a point in the water surface, set perpendicular to the lines of equal velocities and end at right angle on the boundary of the channel. The basic concept is similar to the classical approach of balancing water gravity component and the induced shear force on the boundary for the whole cross section. For the ray-isovel method, the boundary shear stress between any two rays is equal to the downstream component of the weight of water between the rays divided by the wetted perimeter that separates them. Obviously, this approach tends to obtain the variation of the shear stress along the wetted perimeter of straight channel. However, for curved channels the variation of the boundary shear stress in the longitudinal and traverse direction depends largely on the degree of curvature and traverse bed topography corresponding to different location across the bend.

The velocity distribution for rough boundary based on Prandtl- Von-Karman equation is given by

$$\frac{u}{u_s} = 5.75 \log \frac{v}{k_s} + 8.5 \quad \text{[20]}$$

Where \( u \) is a point velocity (m/s) at height \( y \) (m) from the wall, \( u_s \) is the shear velocity (m/s) and \( k_s \) is the roughness height (m).

The average flow velocity \( V \) for wide channel of bed roughness \( k_{she} \) is obtained by integrating the above equation to get

$$\frac{V}{u_{sbe}} = 5.75 \log 12.217 \frac{d}{k_{she}} \quad \text{[21]}$$

From Eq. [20] and [21] one gets

$$\frac{u}{V} = \frac{\log \frac{V}{u_{sbe}}}{\log 12.217 \frac{d}{k_{she}}} + \frac{8.5}{5.75 \log 12.217 \frac{d}{k_{she}}} \quad \text{[22]}$$

Where \( d \) is the flow depth and \( u_{sbe} \) is shear velocity indicating the intensity of turbulence near the bed in the vicinity of the bank.

Fig. 9a which is a plan view demonstrates the distribution of the central and near bank velocities. Fig. 9b demonstrates the situation of the bank where the bottom layer has gone an undercut of a distance \( \Delta L \) at time \( t \). The region influenced by the bank (bank zone) is defined as that part of the channel of width equal to flow depth \( d \). In this region, the velocity distribution corresponding to the eroded layer is as shown in Fig. 9b.
If the bottom eroded layer is of thickness \( L_n \) then at height \( y = L_n/2 \) above the bed the velocity \( u \) is from Eq. [22]

\[
\frac{u}{V} = \frac{\log \left( \frac{L_n/2}{\kappa_{she}} \right)}{\log 12.217} + \frac{8.5}{5.75 \log 12.217 \frac{d}{\kappa_{she}}} \tag{23}
\]

For the bank zone, the velocity distribution (x-z plain) is given by

\[
u = \frac{5.75 \log 12.217 \frac{d}{\kappa_{ba}}} + 8.5
\]

Where \( u_{ba} \) is shear velocity corresponding to the bottom layer of the bank subjected to erosion.

If at any time \( t \), the lateral undercut distance is \( \Delta L \) then for \( x = d + \Delta L \) we have \( v = u \). Substituting into Eq. [24] then

\[
\frac{u_{ba}}{u_{ba}} = \frac{5.75 \log 12.217 \frac{d + \Delta L}{\kappa_{ba}}} + 8.5
\]

From Eq. [23] and Eq. [25] the shear velocity \( u_{ba} \) is obtained as

\[
u_{ba} = C \left[ 5.75 \log 12.217 \frac{d + \Delta L}{\kappa_{ba}} + 8.5 \right]
\]

Where,

\[
C = V \left[ \frac{\log \left( \frac{L_n/2}{\kappa_{she}} \right)}{\log 12.217 \frac{d}{\kappa_{she}}} + \frac{8.5}{5.75 \log 12.217 \frac{d}{\kappa_{she}}} \right]
\]

Accordingly, the induced flow shear \( \tau_{ba} \) on the eroded layer is given as

\[
\tau_{ba} = K \rho \nu_{ba}^2
\]

Where \( K \) a factor its magnitude depends on the degree of curvature of the bend. For straight channels or channel reaches of smaller degree of curvature \( K = 1 \).

Clearly, Eq.[23] to Eq.[28] can further be generalized to obtain the near bank shear stress distribution in order to compute the induced boundary shear force acting on strip or area located over the entire submerged bank slope. However, theoretical analysis and laboratory measurements indicated a diminishing shear stress at the corner of channel section. The corner is defined as the intersection of the bank slope and the channel bed (bank toe area). In other words, the tractive force right at the corner is equal to zero [19-21]. Practically an ideal corner as defined in the analysis or laboratory studies is not achieved in the natural river [22]. Rather, a finite curvature does exist which also exhibits resistance to flow motion at the bank toe zone. Nevertheless, the induced boundary shear stress at the bank toe is less than that given by Eq. [28]. Accordingly, in computing the eroded material the shear stress computed by Eq. [28] need to be adjusted by multiplying the result by a factor say \( \lambda \) possibly less than 0.3 as can be stemmed from the works of Leutheusser [20]. One should stress the fact that further research work is needed to identify the most suitable values of \( \lambda \) for natural channels.

For the River Nile, the eroded bottom layer consists of sand with apparent cohesion. When the layer is wet it loses its cohesion. Mayer-Peter equation or any other suitable bed load transport equation can be applied to compute the amount of eroded sand from area in the vicinity of the toe of the bank. This area includes the bottom layer and the adjacent river bed. The eroded material volume is computed per unit height \( L_n \) of the bottom layer per unit length along the bank base line.

The eroded volume rate \( q_v \) is obtained as

\[
\frac{q_v}{\gamma L_n} = \frac{q_{sh}}{\gamma L_n} = \frac{8}{\gamma_s L} (\lambda \tau_{ba} - \tau_c)^{3/2}
\]

Where, \( Q_v \) is the eroded volume rate of bottom layer (strip of height \( L_n \)). \( \tau_c \) is the critical shear stress for incipient motion given as

\[
\tau_c = \varepsilon \gamma_s D_s
\]

\( \varepsilon \) is a coefficient its value depends on the size of sediment particle and the turbulence intensity. The value of \( \varepsilon \) ranges from 0.03 for rock stones to 0.06 for fine sand. \( D_s \) is the representative particle diameter taken as \( D_{50} \). \( \gamma_s \) is the sediment submerged specific weight.

The eroded volume over a time period \( \Delta t \) is equal to \( \Delta L^* \). Accordingly, the distance \( \Delta L \) is computed as

\[
\Delta L = q_v \Delta t
\]

Computation procedure:

The computation scheme is demonstrated in the flow chart of Fig. 10. Basic input data to the computational model are the river water level (hydrograph), initial bank surface slope geometry, identified soil properties, and predicted or computed tension crack depth (filled with water or not). It is required first to compute the critical shear resistance to erosion \( \tau_c \) for weak layer and the tension crack depth \( y \). If no
tension cracks developed set y=0. The computations start at $t=t_0 + \Delta t$ where at time $t_0$ the bank is assumed stable. From the flow data and sections properties the magnitude of the adjusted induced shear $\lambda \tau_b$ shall be computed and compared with $\tau_c$. If $\lambda \tau_b > \tau_c$, then the bottom layer will subject to erosion. Accordingly, for time duration $\Delta t$ the lateral undercut distance $\Delta L$ and the eroded volume from the bottom layer should be calculated. This step is followed by computation of the driving force $F_D$ and resisting force $F_R$. If the driving force is greater than the resisting force, bank failure will take place and consequently the failure angle $\beta$ and volume of the failed block $\text{BV}$ are identified. In case the resisting force is greater than the driving force no bank failure will occur and the computation process is carried out for the next time step given as $t=t_0 + \sum \Delta t$. For each additional time step $\Delta t$, the flow data is updated and the cumulative value of $\Delta L$ is introduced (Eq. [9]) before starting a new iteration of computation.

Clearly, certain limits are needed to avoid infinite looping and results that are not supported or commonly practiced in the field. It is preferred to set the maximum value of $\beta$ as 90° and the minimum to start the computation as 55°. If $\beta>90^\circ$ then one can have a bank geometry of the shape shown in Fig. 11 (after failure) resulting in a hanging bank. This is not commonly practiced in stratified river banks that prone to bank undercut. In addition, when $\beta=90^\circ$ it is the case corresponds to cantilever failure discussed above. It is advised to set a limiting value for $\Delta L$ as the full height of the bank (i.e. the computation process shall stop when $\Delta L=H_b$). This means that the bank is stable (no failure). Regarding the initial bank surface sloping angle $i$, it is required for vertical bank to set the angle as $i=89^\circ$ to avoid computational difficulties.

The limit on the computation time (duration) is decided from the flow hydrograph. For the Nile it is about 3 months corresponding to the flood period. However, when the flood is less than or recedes to the limit that the induced boundary shear is not large enough to erode the weak bank layer; the computation should stop regardless of the seasonal flood duration.

The application of the model for the Nile River in Sudan in Northern State – reach Merowe – Gennati is discussed below

**River Nile case (Northern Sudan):**

The banks of the River Nile in Sudan consist of cohesive stratified banks resting on sandy bed. The banks being subjected to severe erosion attributed mainly to the presence of a weak or loose stratum underlying its top soil. The following average data is obtained for the reach Merowe – Gennati in Northern State.

Bank height=7m, initial bank angle= 80°,
For the bottom layer $D_{0a}=2\text{mm}$, $s_g=2.65$,
For the bed $D_{0b}=2-3\text{mm}$, $s_g=2.65$.

The layer properties are shown in Table (1) below. For the bends, $k=1.9$, also $\lambda=0.22<0.3$, $e=0.06$.

The hydrograph of Fig (1) is applied for this case.

**Results:**

Computed tension crack depth=1.0m.
At failure:
Undercut distance, $\Delta L=1.47m$,
Failure block width, $BW=2.63m$,
Failure angle, $\beta=64.1^\circ$.
Failure block volume =12.87m$^3$.

**Table 2** hereafter gives the bank status as flood discharge builds up. The computations were carried out for time step of one day. The computation stopped when bank failure took place on the 26th day from the beginning of the flood. This is in a good agreement with field observations where the bank failure commonly occurs at the early stage of flood rising. However, the failure block width is little larger than the commonly observed one which ranges from 1.5m to 2m for the reach considered.

**Table 1. Geotechnical properties of the bank layers**

<table>
<thead>
<tr>
<th>Layer position</th>
<th>Sp. Weight (kN/m$^3$)</th>
<th>Cohesion (kN/m$^2$)</th>
<th>$\Phi$ (Deg)</th>
<th>Depth thick. (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>17.5</td>
<td>15.5</td>
<td>27</td>
<td>1.0</td>
</tr>
<tr>
<td>Second</td>
<td>18.2</td>
<td>12.0</td>
<td>25</td>
<td>2.0</td>
</tr>
<tr>
<td>Third</td>
<td>18.5</td>
<td>11.8</td>
<td>29</td>
<td>3.0</td>
</tr>
<tr>
<td>bottom</td>
<td>22.5</td>
<td>1.8</td>
<td>27</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Table 2. Nile bank status as the flood discharge builds up**

<table>
<thead>
<tr>
<th>Flood period (Day)</th>
<th>discharge (cms)</th>
<th>depth (m)</th>
<th>width (m)</th>
<th>$\Delta L$ (m)</th>
<th>Bank status</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10</td>
<td>1160</td>
<td>3.8</td>
<td>400</td>
<td>0.0</td>
<td>stable</td>
</tr>
<tr>
<td>11 - 16</td>
<td>1620</td>
<td>4.1</td>
<td>455</td>
<td>0.06</td>
<td>stable</td>
</tr>
<tr>
<td>17 - 24</td>
<td>2315</td>
<td>4.4</td>
<td>465</td>
<td>1.17</td>
<td>stable</td>
</tr>
<tr>
<td>25 – 26 - 29</td>
<td>3240</td>
<td>4.9</td>
<td>480</td>
<td>1.47</td>
<td>failure</td>
</tr>
</tbody>
</table>
Input: bank geometry, bank soil, flow hydrograph, section properties, limits, Δt

START

Compute: $\tau_{ba}$

$\tau_{ba} > \tau_c$?

Yes

Compute: $q_V, \delta L, \Delta L$

Compute: $\beta, F_{h1}, F_{h2}, F_B, W$

$\beta \geq 90^\circ$ or $\Delta L \geq H_o$?

No

Limit hydrograph? Or time limit?

No

Compute: $F_D, F_R$

$F_D \geq F_R$?

No

Yes

Bank Failure

Output: $\Delta L, \beta$

Compute: $\beta$, $\Delta L$, $BW$, $BV$

Output: $\beta, \Delta L, BW, BV$

STOP

Bank stable

Output: $\Delta L$

Yes

Limit hydrograph? Or time limit?

No

Yes

STOP

Output: $\Delta L$

STOP

Fig.10. Flow Chart of computation
3. SUMMARY AND CONCLUSIONS

In this paper river bank erosion due to the process of undercutting is analyzed. Bank failure is attributed to presence of weak erodible layer at or near the river bed level. The layer normally consists mainly of sand with low percentage of silt and clay. The layers on top of this layer consist of cohesive soil material which provides large resistance to erosion caused by the induced flow shear force on the bank. When the weak layer is attacked by strong flow current the resulting induced flow shear on the bank erodes the layer (undercut). The lateral undercut distance through this layer leads to bank failure when it exceeds the critical value. For steep banks failure surface is of a planner shape. The failure block slides over the plain surface and sometimes it just rotates about the lowest point in the failure surface.

The lateral undercut distance value ∆L at which the bank fails is determined by integrating soil mechanics theories, basic hydraulics principles and hydraulics of sediment transport of loose boundary channels. The induced hydraulic forces, hydrostatic pressure force, buoyant force, gravity forces and bank soil resistance are combined to develop a model that simulates stratified bank failure due to undercutting. The model stability analysis takes into consideration the effect of tension cracks whether dry or filled with water. In order to determine the shear force acting on the bank, it is required first to determine near-bank velocity distribution. The near-bank velocity distribution was developed based on mixing length theory applied to rough boundary. The magnitude of the shear force acting on the bottom layer was determined as a function of flow depth and shear velocity. Using Meyer-Peter-Muller equation, the rate of the eroded amount of sediment was determined and hence the undercut distance ∆L and the lateral rate of shift (migration) of the bank once failed for the time period considered.

A computation scheme to determine the lateral undercut distance was demonstrated. Limitations on the computation process were also identified. The computation should proceed for the identified flood period as long as the adjusted induced boundary shear stress is greater than the critical shear resistance to erosion of the soil particles of the bottom layer. Application of the model is demonstrated for the River Nile bank erosion in Northern Sudan State and the agreement was found to be good.

In conclusion, river bank erosion due to undercutting is a complicated phenomenon which requires further investigation and analysis that considers in depth the variability of the bank soil properties, flow parameters, and morpho-dynamic of the river reaches.

REFERENCES


Coastal Engineering, Technical University of Hamburg, Germany


NOMENCLATURE

FS = factor of safety
FR = resisting force
FD = driving force
LL= length of the failure plane EF (later called EO)
N= Force component normal to the failure plane EF
C = Effective soil cohesion
Φ = effective internal angle of friction
L = layer thickness
N = number of layers
γ, γi = soil specific weight (dry bank)
i = bank inclination angle
H = bank height
L = thickness of n layer (bottom layer)
W = self weight of the failure block
FB = buoyant force
Fh= hydrostatic force river side
Fh=hydrostatic force due to water in tension crack
ΔL = lateral undercut distance ΔL (at failure of bank)
wj = weight of the jth stratum per unit channel length
γj = weight density of material of the jth stratum.
m= mth stratum at which the tension crack depth ends
γw = specific weight of water
θ = average value of θ or a preselected value
BV = volume of failure block
ΔL = lateral eroded distance
BW = failure block width
y = tension crack depth
β = angle of the failure plane EO.

δΔ = incremental value of the under cutting distance time step
Δt = other words
t = time to failure
k = required number of time steps to arrive at failure.
W = weight of failure block per unit channel length
ξ = jth stratum bottom width in the presence of tension cracks
h = water depth in tension crack
u = point velocity at height y from the wall (central flow)
u = shear velocity
k = roughness height
V = average flow velocity
k = channel bed roughness
d = average flow depth
u = point velocity falls in the boundary between central and bank flow
v = point velocity at distance x (x-z plain)
ω = shear velocity related to bank bottom layer.
τn = induced flow shear on the eroded layer
K = bend degree of curvature factor
q = eroded volume rate per unit height L
Q = submerged weight of eroded material per unit time
Q = eroded volume rate of bottom layer
τc = critical shear stress for incipient motion
ε = turbulence intensity coefficient.
D = representative particle diameter.
γ = sediment submerged specific weight.
λ = sediment stress adjustment factor