



Finite Element Analysis of Reinforced Concrete Beams Strengthened with Carbon Fiber Reinforced Polymer Sheets

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ABSTRACT

This paper presents outcomes of extensive research conducted to study the behavior of reinforced concrete (RC) beams strengthened with CFRP sheets and subjected to shear and flexural loading. A 3-dimensional Finite Element (FE) modelling was carried out using ABAQUS software program to simulate the flexural and shearing behavior of a number of RC beams strengthen with CFRP. Moreover, the results of the numerical simulation were compared with actual results of experimental physical modelling of RC beams subjected to the same loading conditions for validation purpose. Appreciable number of sets of control RC beams and CFRP strengthened RC beams for flexural and shear were investigated. All beams were rectangular in cross section. To accurately describe the behavior of these tested RC beams, different material constitutive models were used in the simulation. Concrete damaged plasticity model was used to model concrete; a linear elastic perfectly plastic model was used to model both reinforcing steel and stirrups, while a linear elastic isotropic model was used to simulate the behavior of CFRP material. Moreover, the interface between concrete and CFRP material was accounted for and was simulated using a tie perfect bond model while steel reinforcement is bonded with concrete as an embedded element in ABAQUS. A careful comparison between the results of the experimental tests and those of the numerical simulation showed that the proposed FE model is able to describe the behavior of reinforced concrete beams strengthened with CFRP in terms of crack pattern and load

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deflection curves to an appreciable level of accuracy. Generally, the results demonstrate that numerical approach can be used to deeply investigate the behavior of RC beams strengthened with CFRP sheets. It was also concluded that the numerical approach was able to capture significant aspects of structural behavior of the tested beams such as strain distribution in CFRP, which is often not easily captured experimentally or analytically.

المستخلص

في هذه الدراسة، تم إجراء تحليل عددي باستخدام برنامج تحليل العناصر المحددة (ABAQUS) بهدف دراسة سلوك الانحناء والقصر للعتبات الخرسانية المدعمة بالألياف المسلحة بالبوليمرات. بالإضافة إلى ذلك، تم مقارنة نتائج المحاكاة مع تلك البيانات التي تم الحصول عليها من النتائج العملية للتحقق من نموذج العناصر المحددة. تم التحقق من العتبات الخرسانية الغير مدعمة والمدعمة بالألياف المسلحة بالبوليمرات للثني والقصر. حيث أن جميع العتبات مقطوعها العرضي مستطيل. لوصف سلوك هذه العتبات الخرسانية، استخدمت نماذج مختلفة من المواد. تم استخدام نموذج الخرسانة اللدنة التالفة (CDP) للخرسانة، نموذج بلاستيكي مرن خطي تماماً استخدم لكل من حديد التسليح والكانات، بينما تم استخدام نموذج خطي مرن للألياف المسلحة بالبوليمرات. تم نمذجة الارتباط بين الخرسانة والألياف المسلحة بالبوليمرات بربط كامل مثالي بينما تم نمذجة الترابط بين حديد التسليح والخرسانة كعنصر جزء لا يتجزأ في برنامج ABAQUS. أظهرت نتائج التحليل العددي أن نموذج العناصر المحددة المقترح قادر على وصف سلوك العتبات الخرسانية المسلحة المدعمة بالألياف المسلحة بالبوليمرات (FRP) من حيث أنماط التشققات ومنحنيات الحمل والانحراف، مقارنة بالنتائج العملية التي تم الحصول عليها من نفس العينات. عموماً، تظهر النتائج أن التحليل العددي يمكن استخدامه للتحقق بعمق في فهم سلوك العتبات الخرسانية المدعمة بالألياف المسلحة بالبوليمرات (FRP). ولوحظ أيضاً أن هذه الطريقة قادرة على الحصول على توزيع الانفعالات في الألياف المسلحة بالبوليمرات (FRP)، والتي لا يسهل الحصول عليها عملياً أو تحليلياً.

Keywords: reinforced concrete beams; CFRP; flexural behavior; shear behavior; finite element method

1 Introduction

Although hundreds of thousands of successful reinforced concrete structures are annually constructed worldwide, there are large numbers of concrete structures that deteriorate or become unsafe due to inadequacy of design detailing, construction and lack of adequate maintenance,

overloading, chemical attacks, corrosion of rebar, foundation settlement, abrasion, fatigue effects, atmospheric effects, changes in use, changes in configuration, and natural disasters such as earthquake (Jumaat et al. 2006).

In modern structural engineering, carbon fiber reinforced polymers are very common solutions for strengthening and retrofitting of deficient concrete structures, *Lima* (2016). Due to its inherent advantages including light weight which makes it easy to handle, high strength about five times higher than structural steel, and corrosion resistance which increases useful lifetime of structures and is also highly versatile in application. The most common configurations include side wrap, U-shaped and full wrapping for shear strengthening and bottom wrap is most common for flexural strengthening and retrofitting (*Panjehpour, Aliet et al. 2014, Premet et al. 2016*).

May be due to the fact that accurate modelling of concrete shear and flexural cracking and interfacial interactions between concrete and internal steel and with CFRP and concrete is truly of challenging nature, very limited research work on detailed finite element modelling of shear and flexural critical beams is available in the literature. This paper presents results of an appreciable course of research, which led to a Master's degree conducted to study flexural and shear behavior of reinforced concrete beams strengthened with CFRP sheets, (Ahmed, Daoud 2018). In this research, Abaqus/standard, finite element software program was used to simulate the behavior of the concrete beams strengthened with CFRP sheets. This FEM package includes a large variety of material models and elements including facilities necessary for this particular subject. This involves several aspects of theoretical and practical interest. Important issues include material models, element types, mesh, convergence and boundary conditions. A general conclusion regarding these issues is that the model must be rich enough to be able to capture the important phenomena, but it should not be more complex than necessary since this would only increase the computer time needed. To be able to verify the quality of the model, results was compared to experimental results.

In this paper, an advanced finite element models to simulate cracking in concrete are developed, discussed and adopted. The models take into account the numerical computational difficulties, convergence and solution degradation problems associated with other concrete models. Modelling of concrete, reinforcing steel and CFRP and concrete-steel and concrete-CFRP bond and contact behaviors are then discussed.

Finally, the commercially available non-linear finite element software ABAQUS is used to validate the developed finite element model with key tests performed on full scale rectangular-beams with and without CFRP retrofiting. The validation was based on the extensive experimental study “flexure and shear strengthening of Reinforced Concrete Beams Using Carbon Fiber Reinforced Polymers Sheet” by *Nadeem A. Siddiqui* (2009) and *Tirab, UOK*, (2015).

2 Finite Element Model

Finite element model is presented below delineating the modelling to simulate flexural and shear cracking in concrete followed by modelling of reinforcing steel and CFRP and their contact behavior with concrete. Displacement control loading is applied and dynamic explicit solution approach is adopted which has a conditionally stable solution technique using explicit integration. This type of analysis was chosen because it showed stability in solving problems that are static and Quasi-static process modelling problems including complex contact (*Simulia* 2013). This method can be good to simulate material degradation or failure, such as cracking of concrete. Based on the literature, analysis using this method rarely encounters problems of convergence. However, for static problems using dynamic analysis, inertial effects should be minimized by using slow loading rates or by increasing the mass density so that the oscillation of the results is limited (*Mercan* 2011). In this study, it has been found that using loading time one second has given good results through defining two load steps (initial and dynamic explicit) with automatic increments and unlimited maximum time increments. The nonlinear finite element software ABAQUS is utilized to conduct the study and the proposed modeling parameters.

2.1. Cracking Simulation in Concrete

To simulate quasi-brittle nature of reinforced concrete, various conceptual models are available in literature which includes discrete crack model, smeared crack model, and inner softening band. Numerical modelling of concrete cracking is mostly carried out using discrete crack and smeared crack models such as the one proposed by Chen GM (2010). In the discrete crack model, opening which arises due to cracks are physically modelled and taken as a geometrical identity. Since cracks are defined along the surfaces of the elements, they create mesh bias. A lot of researches attempted to resolve this issue by developing finite element codes capable of generating re-meshing like Yang (2003, 2004, 2005) but the computational difficulties associated with re-meshing is still a great challenge as found by De Borst, Remmers et al. (2004). While in the case of smeared crack models, a phenomenon known as “strain localization” leads to zero energy utilization during crack opening when element size approaches zero, this results in a non-mesh objective case that causes solution not to converge or material to degrade.

In this paper, damage plasticity model has been utilized for concrete which is a constitutive model available in the ABAQUS material library. In the damage plasticity model, compression and tension are two hardening variables that control development of yield surface. A continuum damage mechanism is used to model crack opening and its propagation by stiffness degradation approach which essentially means that elasticity is degraded in concrete where it cracks.

2.1 Concrete Model

To model a concrete material in ABAQUS “Concrete Damaged Plasticity” model was developed by (Lubliner, Oliver et al. 1989) and elaborated by (Lee and Fenves 1998). The model is used which requires the following material functions:

- Uniaxial stress-strain relation of concrete under compressive and tensile loading.
- Damage parameters dc and dt for compressive and tensile load, respectively. These parameters could be used to identify and

validate damage pattern of the developed model and compare it with experimental work.

The stress-strain behavior under sustained compressive loading is modelled in three phases (Figure 1).

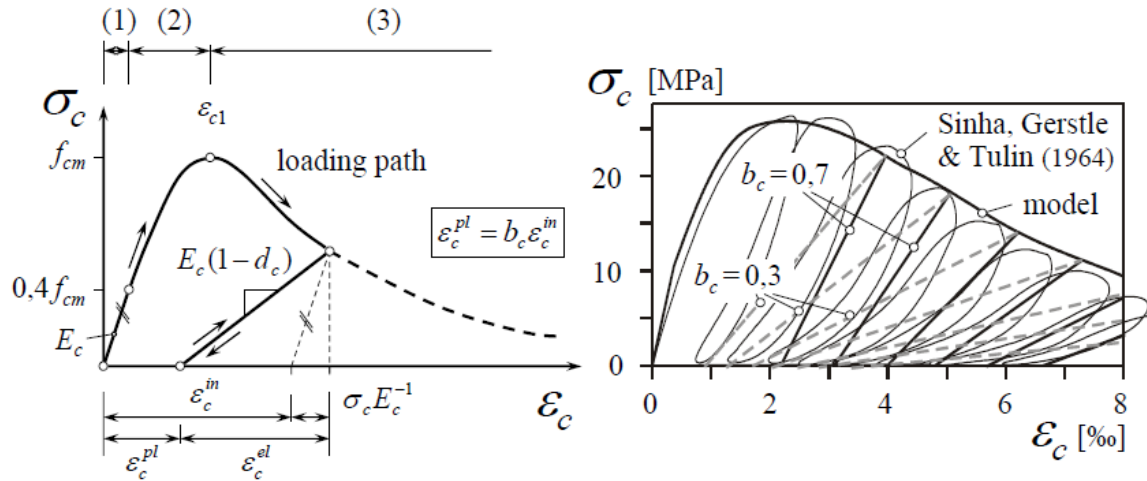


Figure 1: Stress-strain relation for (cyclic) compressive loading (Sinha et al., 1964)

$$\sigma_{c(1)} = E_c \varepsilon_c \tag{1}$$

$$\sigma_{c(2)} = \frac{E_{ci} \frac{\varepsilon_c}{f_{cm}} - (\varepsilon_c / \varepsilon_{c1})^2}{1 + (E_{ci} \frac{\varepsilon_{c1}}{f_{cm}} - 2) \frac{\varepsilon_c}{\varepsilon_{c1}}} f_{cm} \tag{2}$$

$$\sigma_{c(3)} = \left(\frac{2 + \gamma_c f_{cm} \varepsilon_{c1}}{2 f_{cm}} - \gamma_c \varepsilon_c + \frac{\gamma_c \varepsilon_c^2}{2 \varepsilon_{c1}} \right)^{-1} \tag{3}$$

The first two sections describe the ascending branch up to the peak load f_{cm} at ε_{c1} . Their formulations are similar to the recommendations of the Model Code (CEB-FIB, 1993). The third and descending branch takes account for its dependency on the specimen geometry (Vonk, 1993; Van Mier, 1984) to ensure almost mesh independent simulation results. Thus, $\sigma_{c(3)}$ incorporates within the descent function γ_c the constant crushing energy G_{c1} (Krätzig & Pölling, 2004) as a material property in addition to an internal length parameter l_c derived from the grid structure of the element mesh.

$$d_c = 1 - \frac{\sigma_c E_c^{-1}}{\varepsilon_c^{pl} (1/b_c - 1) + \sigma_c E_c^{-1}} \tag{4}$$

The evolution of the compressive damage component d_c is linked to the corresponding plastic strain ε_c^{pl} which is determined proportional to the inelastic strain $\varepsilon_c^{in} = \varepsilon_c - \sigma_c E_c^{-1}$ using a constant factor b_c with $0 < b_c \leq 1$.

A value $b_c = 0,7$ fits well with experimental data of cyclic tests (Figure 1, right). So, most of the inelastic compressive strain maintains after unloading. Generally, unloading and subsequent reloading up to the monotonic path occur linearly with no hysteretic loops.

The stress-strain relation $\sigma_t(\varepsilon_t)$ for tensile loading consists of a linear part up the strength f_{ct} and a nonlinearly descending part that depends on the specimen geometry (Figure 2). The latter function is derived from the stress-crack opening relation (Hordijk, 1992).

$$\frac{\sigma_t(w)}{f_{ct}} = \left[1 + (c_1 w/w_c)^3\right] e^{-c_2 \frac{w}{w_c}} - \frac{w}{w_c} (1 + c_1^3) e^{-c_2} \quad c_1 = 3, c_2 = 6,93 \quad (5)$$

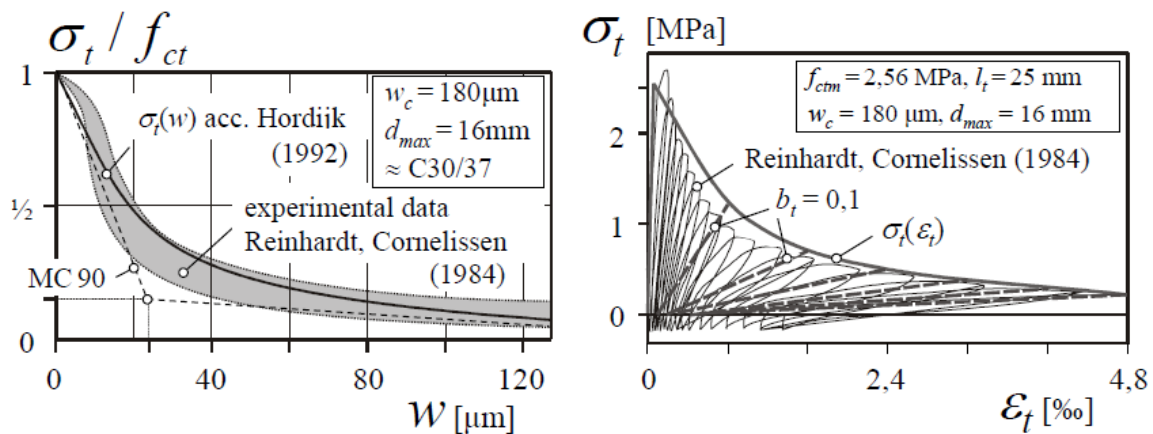


Figure 2: Stress-crack opening and stress-strain relations for (cyclic) tensile loading, experiments acc. (Reinhardt & Cornelissen, 1984).

Using the principle of the "Fictitious Crack Model" (Hillerborg, 1983). Thus, a product of the inelastic strain and an internal length parameter l_t replaces the crack opening w to yield $\sigma_t = \sigma_t(w = l_t \varepsilon_t^{in} = l_t (\varepsilon_t - \sigma_t E_c^{-1}))$ and w is smeared over the average element length $l_t = V_e^{1/3}$. As intended, $\sigma_t(\varepsilon_t)$ then encloses the ratio of fracture energy G_F and l_t (Bazant & Oh, 1983).

Similar to (4) the damage d_t depends on ε_t^{pl} and an experimentally determined parameter $b_t = 0,1$ (Figure 2, right). So, unloading is assumed to return to the origin and to leave only a small residual strain.

$$d_t = 1 - \frac{\sigma_t E_c^{-1}}{\varepsilon_t^{pl} (1/b_t - 1) + \sigma_t E_c^{-1}} \quad (6)$$

Table 1 below shows the recommended values for CDP model other parameters of for normal concrete.

The expansion angle (ψ) and eccentricity (ϵ) are parameters related to the yield surface flow rule. (K) is the parameter that controls the shape of the yield surface. (f_{bo}) is the concrete biaxial compressive strength, (f_{co}) is the uniaxial compressive strength, and the ratio of the two is taken as the recommended

value. (μ) is the viscous parameter defined by the CDP model. The larger (μ) is, the easier it is to converge. The smaller (μ) is, the higher the accuracy is. When (μ) is 0.01, reasonable accuracy and convergence can be satisfied.

Table 1: Concrete parameters used in the plastic damage model

Concrete Strength (MPa)	Dilation Angle (ψ)	Eccentricity (ϵ)	f_{bo}/f_{co}	K	Viscosity parameter (μ)
Varies	36°	0.1	1.16	0.67	0.01

2.2 Modeling of Reinforcing Steel and the Bond with Concrete

Reinforcing steel has been modelled using a 2-noded linear 3-D truss, the stress-strain curve adopted for steel is elastic-perfectly plastic, the other parameters used to define reinforcing steel behavior are shown in Table 2.

Table 2: Parameters of reinforcing steel

Property	Value
Elastic Modulus (MPa)	200,000
Poisson's ration	0.3
Mass Density (tone/mm3)	7.85E-009
Yield Stress (MPa)	535.4/634.8

2.3 Modeling of Reinforcing Steel and the Bond with Concrete

Steel reinforcement is bonded with concrete as an embedded element in ABAQUS. Embedment technique successfully used by many researchers

and considered a very powerful finite element tool which allows number of elements to be embedded inside another element which known as host element. One of the significant utilizations of embedment technique is the modelling of interaction surface between the embedded and the host element, which eradicates numerical costly iterations linked with surface formulations.

2.4 Modeling of CFRP and the Bond with Concrete

CFRP has been modelled as a composite lamina in ABAQUS. The properties of CFRP used to model the lamina as linear elastic material are shown in Table 3.

The interaction between CFRP and concrete is considered as Tie (perfect) bond since the information about the adhesion materials is not available. That define the bond interaction and reflects the behavior of adhesive material between concrete and the rest of CFRP strip, a default contact enforcement method in ABAQUS was adopted.

This assumption has been validated with study of different geometric and material parameters of the beams under consideration.

Table 3: properties of CFRP used to model the lamina based on Nadeem A. Siddiqui (2009)

Properties	Value
Ultimate tensile strength f_{fu}^*	846 N/mm ²
Rupture strain ϵ_{fu}^*	0.011 mm/mm
Modulus of elasticity of CFRP laminates E_f	77.28 N/mm ²
Thickness t_f	1.0 mm

3 Empirical Models

Two groups of full scale experimentally tested beams are chosen from study on “efficiency and effectiveness of CFRP schemes for flexure and shear strengthening of RC beams” by Nadeem A. Siddiqui (2009). For this purpose, four RC beams were cast in two groups, each group containing two beams. All the beams had a cross section of 200 × 300 mm, a simply supported span of 2000 mm. Out of a total number of two beams, one beam in each group used as a control specimen, and other two employed to prepare

CFRP strengthened beams. (Figure 3 and Figure 4) shows the details of beam specimens.

The experimental RC beams in Table 4, Figure 5, and Figure 6 are taken for the comparison of crack pattern with the analytical models using finite element software (ABAQUS).

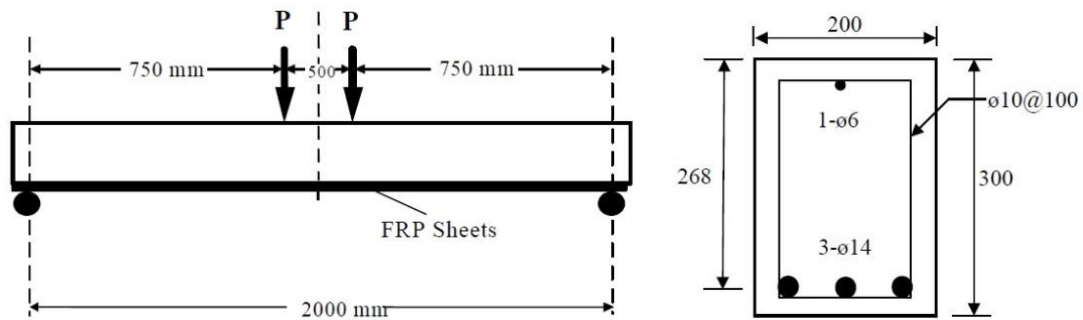


Figure 3: Cross section of Group-1 beam specimens strengthened under scheme-1.

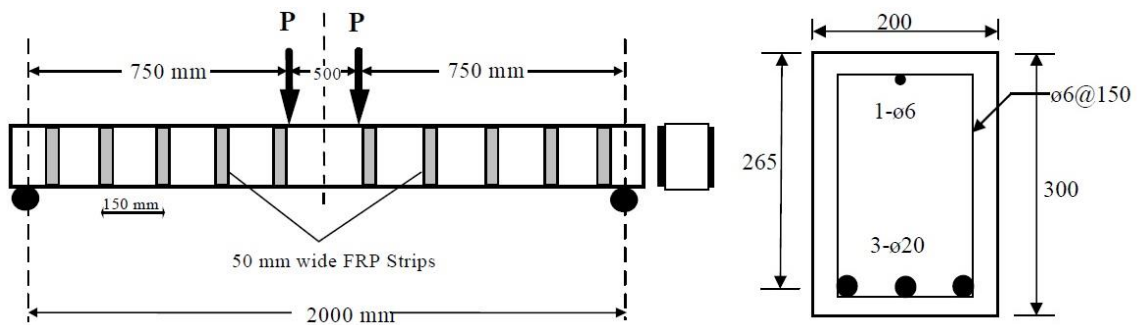


Figure 4: Cross section of Group-2 beam specimens strengthened under scheme-2.

Table 4: Details of test specimens (Nadeem A. Siddiqui 2009)

Group No.	Specimen Designation	Details of Strengthening Schemes
1	BCF	Control beam
	BFS-1	Bottom strengthening for flexure
	BCS	Control beam
2	BSS-1	Vertical CFRP strips strengthening for shear



**Figure 5: Beam strengthened with CFRP sheet for flexural (Group-1).
(Nadeem A. Siddiqui 2009)**



Figure 6: Beams strengthened with vertical strips (Group-2). (Nadeem A. Siddiqui 2009)

4 Results Analysis and Discussions

The proposed finite element model has been validated with full-scale experimentally tested beams, which includes both control and retrofitted beam specimens. A comparison is presented next to validate and calibrate the competency of FEM to envisage the mode of failure, load-deflection curve and overall behavior of control and retrofitted beams.

4.1 Cracking behavior

It is well known that when reinforced concrete beams are subjected to bending, they are prone to cracking. Depending on the shear span ratio, we may expect flexural cracks, and or flexural shear cracks or even purely shear cracks. Cracking in reinforced concrete beams is initiated when the maximum principal stress exceeds the tensile strength of concrete. This is justified by the tensile stress distribution shown in Figure 7 of the control beam (Group-1, Nadeem A. S 2009). The maximum averaged tensile stress displayed was 3.6 MPa, which exceeds the tensile strength of concrete 3.5 MPa. This implies crack initiation at the first increment of load application.

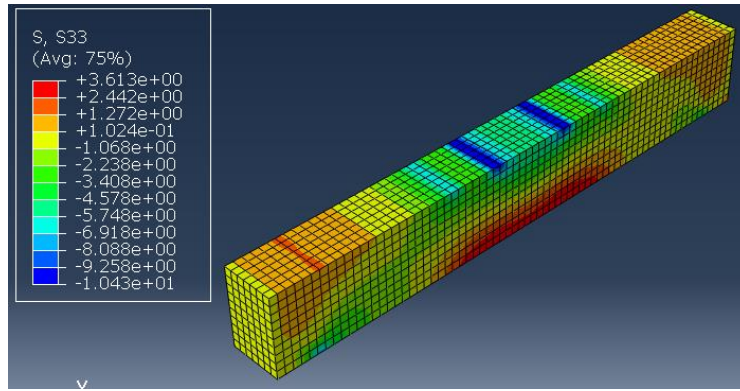


Figure 7: Tensile stress distribution in control concrete beam Group-1, (Nadeem A.S Specimens).

Crack initiation always took place in the middle from the tension face of the beam and extended to the neutral surface; however, the propagation pattern was different for control and strengthened beams. As concrete was cracking, stresses in tension steel were increasing up to yield stress, Figure 8.

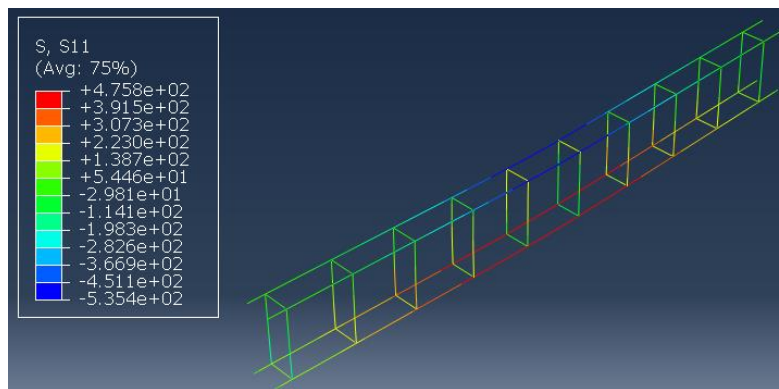


Figure 8: Stress distribution in steel reinforcement (1800mm-Patched RC beam)

It has been observed in this study that for proper crack pattern representation, it is useful to use force control. However, force control did not capture in a perfect manner the load displacement relationship and other failure mechanisms. This was because force control fails to capture the post peak behavior of the beams. Up to the ultimate load, all cracks will have formed before convergence issues arise, i.e. before the stiffness matrix becomes singular. This may be caused by numerical round off in the finite element models where bodies, which were supposed to be in contact, are not. This will result in rigid body motion. On the other hand, displacement control did a better representation of the load displacement relation, as it is able to consider the descending part up to complete failure. Displacement

control failed to represent adequately the crack pattern but helps stabilize the solution. In displacement control, it is possible to have different values of displacement corresponding to one value of force, leading to few cracks as compared to force control. Thus, the crack pattern shown below was derived from force control and the remaining of the output was based on displacement control.

4.2 Deflection Behavior

One of the parameters used to characterize the behavior of reinforced concrete beams subjected to bending is the load-deflection relationship. The relationship is able to represent different stages of failure as the applied load increases. Generally, those stages include crack initiation, yielding of tensile steel reinforcement and crushing of concrete in compression.

In the present study as stated before displacement control was applied and force was requested as history output at the supports. Displacement was also requested in the middle of the span. Figure 9 shows a comparison of the load deflection relations for all the beams analyzed in this study.

From Figure 10 and Figure 11, it is observed that for control RC beam after yielding of steel no further increment in load carrying capacity was experienced until failure. However, for CFRP strengthened beams, this was not the case. This shows that strengthening increase the load carrying capacity of reinforced concrete beams.

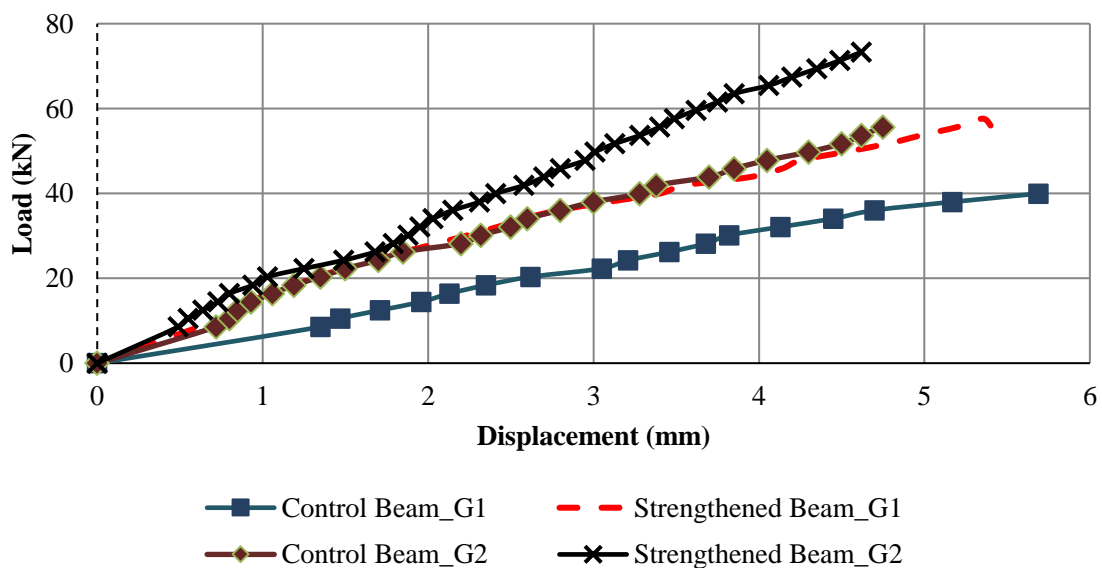


Figure 9: Comparison of FE load deflection for all specimens

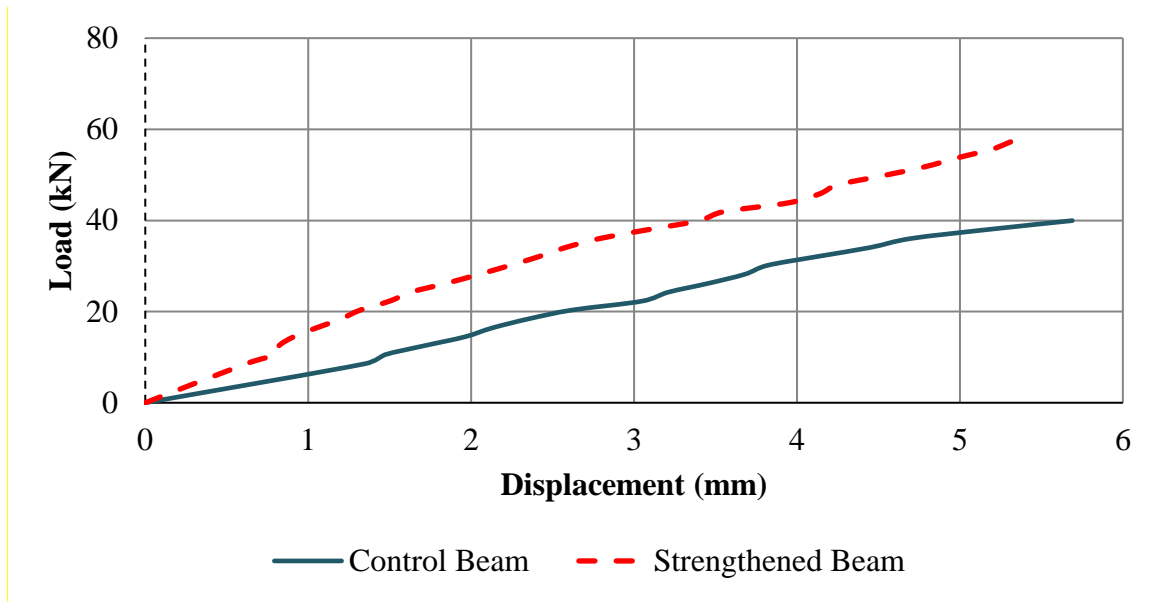


Figure 10: Comparison of analytical load deflection curve of control beam and strengthened beam for flexural strengthening (Group-1)

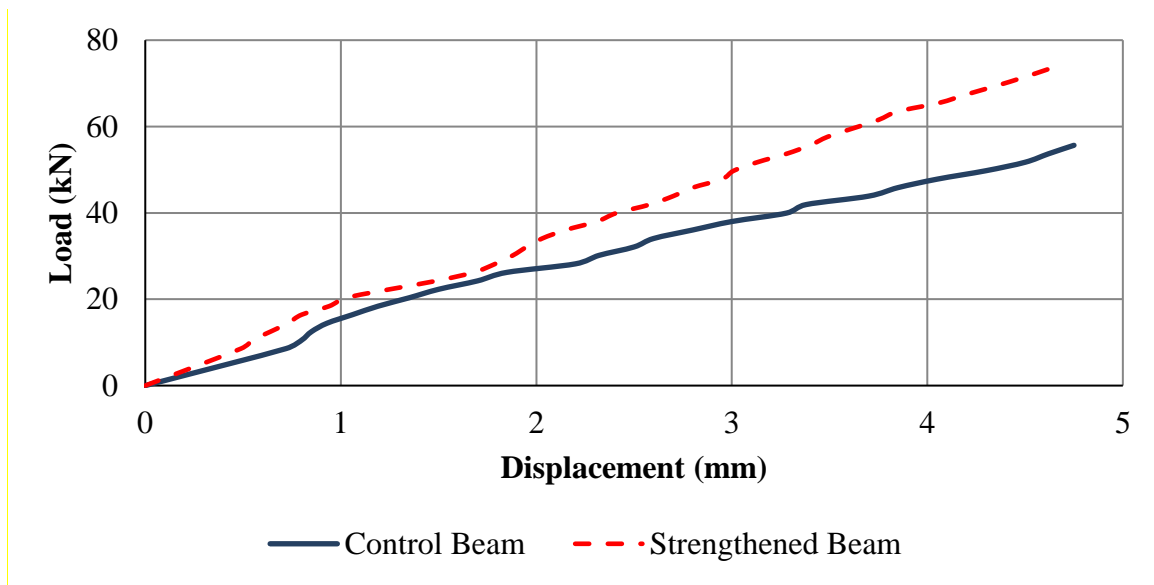


Figure 11: Comparison of analytical load deflection curve of control beam and strengthened beam for shear strengthening (Group-2)

4.3 Validation of the Finite Element Models.

In order to assess the integrity of the finite element model developed in this research thesis and its capability to capture the behavior of reinforced concrete beams strengthened with fiber reinforced polymer materials, two parameters were chosen, load displacement relationships and crack distribution along with de-bonding as observed in the laboratory.

4.3.1 Load Deflection Curve at Mid-Span

The results from finite element analysis were calculated at the same locations as that of the experimental test of the beams. Load deflection curves obtained from both FEM analysis and laboratory work for control beam and strengthened beams were compared as it can be seen in Figures 12, 13, 14 and 15. The validity of the FEM results was assessed by comparison with the experimental results.

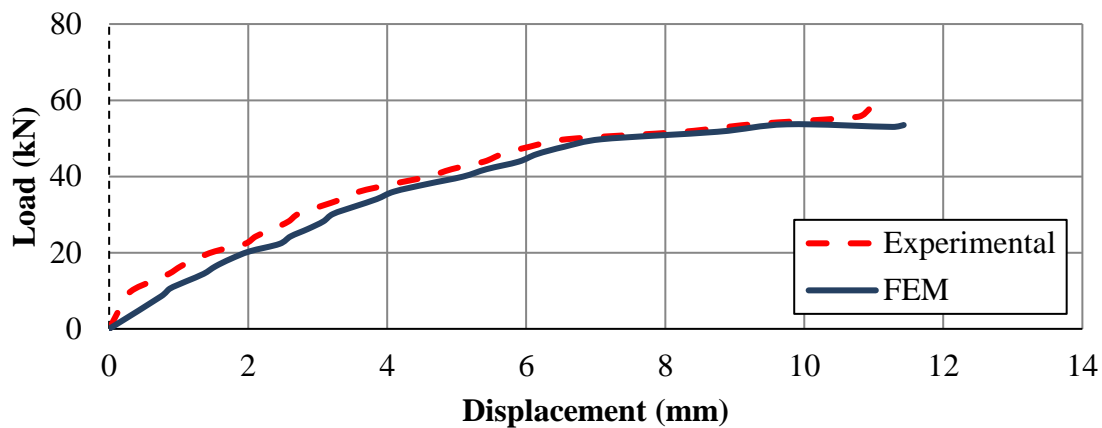


Figure 12: Comparison of analytical and experimental load deflection curve of control beam (Group-1)

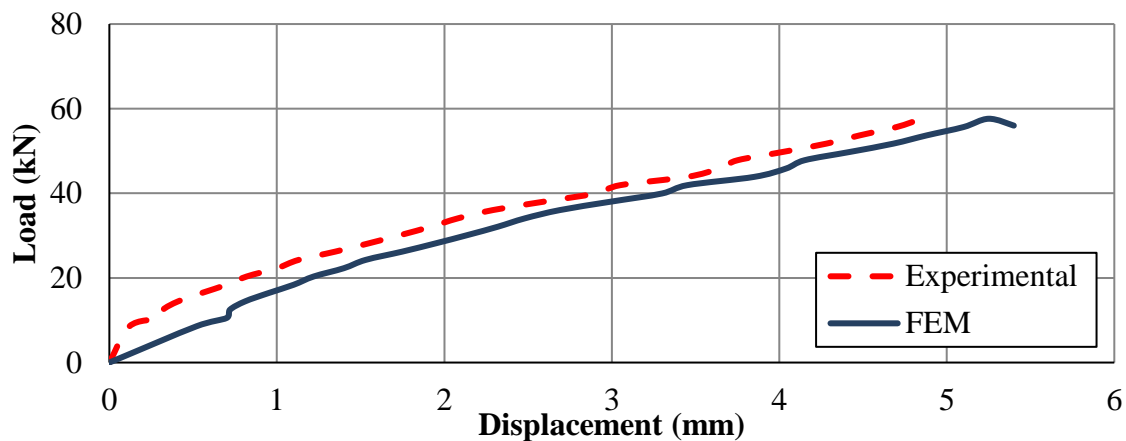


Figure 13: Comparison of analytical and experimental load deflection curve of strengthening beam (Group-1)

The comparison between experimental and the numerical load–deflection curves for the mid span deflection shows that the finite element load deflection curves are somewhat stiffer than the experimental plots. The stiffness of the finite element models is again higher than that of the experimental beams. There are several effects that may cause the higher

stiffness in the finite element models. The most important is micro cracks, which are present in the concrete for the experimental beams and could be produced by drying shrinkage in the concrete and/or handling of the beams. On the other hand, the finite element models do not include the micro cracks (Ibrahim M. Metwally 2014).

Those differences may also due to the assumptions made in the finite element formulation and the fact that beams were not anchored as in experiment. In addition, as reported by Chaudhari and Cakrabarti (2012), concrete damaged plasticity overestimates the stresses in concrete and this could also be a reason of discrepancies.

Generally, its shows that concrete damaged plasticity model in ABAQUS is able to model the overall behavior of reinforced concrete beams strengthened with CFRP materials bonded at their interface.

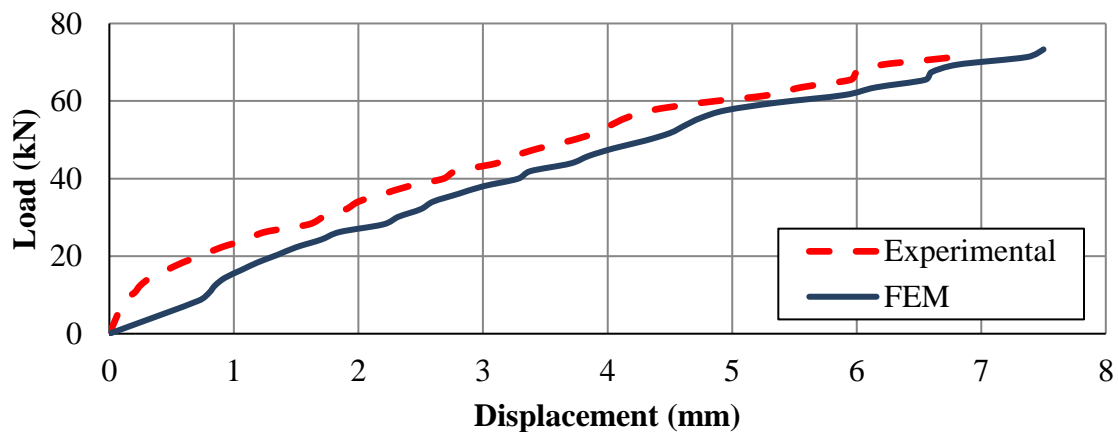


Figure 14: Comparison of FE and experimental load deflection curve of control beam (Group-2)

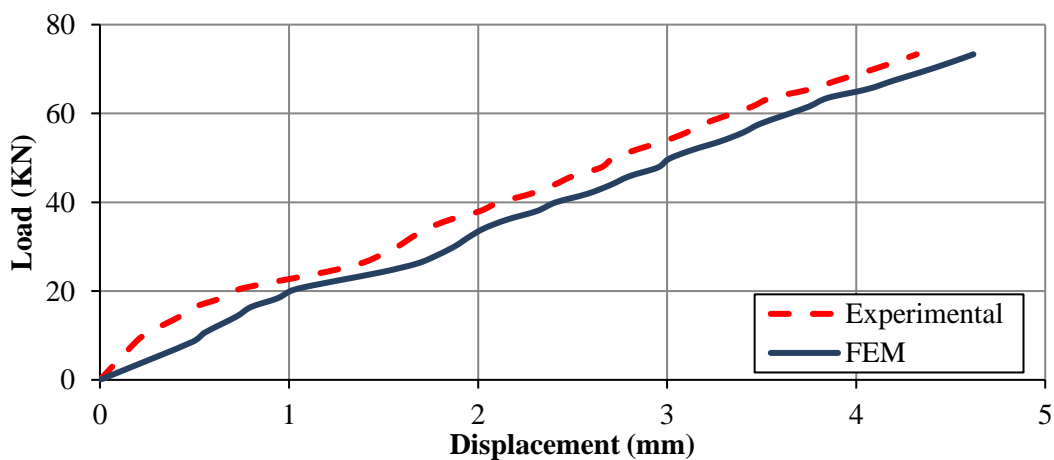


Figure 15: Comparison of FE and experimental load deflection curve of strengthening beam (Group-2)

4.3.2 Crack Distribution and Failure Mechanisms.

Comparison of crack pattern for control beam (group-1) between experiments and FE analysis are shown in Figure 16 and Figure 17.



Figure 16: Crack pattern for control beam, group-1_ (Nadeem A.S 2009) (Experiments)

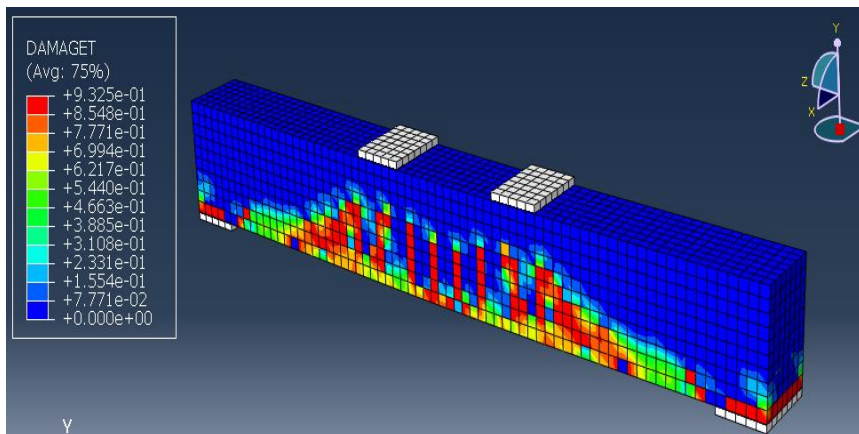


Figure 17: Crack pattern for control beam, group-1 (FE Analysis).

Even though concrete damaged plasticity model did not capture all the cracks observed in the experiments, figures 16 and 17 show that the general pattern of crack in both finite elements analysis and experiments is the same. It is clear that flexural cracks are vertical and flexural/shear cracks are inclined towards loading points. Such differences are due to nonhomogeneous distribution of aggregates in concrete that causes cracks such as the one shown in Figure 18 that could not be captured in numerical analysis. However, these facts do not prevent numerical analysis to be a powerful and reliable tool of analysis as illustrated by the comparison of crack pattern and other aspects between numerical and experimental investigations.

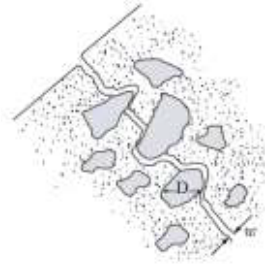


Figure 18: Expected crack path in concrete (Baldvin, 2011)

Furthermore, there are such cracks due to shrinkage, concrete settlement or poor placement of concrete. This shows that even if concrete damaged plasticity model does not have direct option to consider the transition interface zone and other microstructure properties of concrete, it captures in an excellent manner the overall behavior of reinforced concrete.

For the case of CFRP-strengthened reinforced concrete beams for flexural (group-1), Figure 19 and Figure 20 compares finite element and experiments on beam. From the figures below, it was clear from comparison that the location of critical cracks from FEA where debonding took place was corresponding to experimental observations. This once again shows the integrity of the proposed model to study the behavior of reinforced concrete strengthened with CFRP laminate.

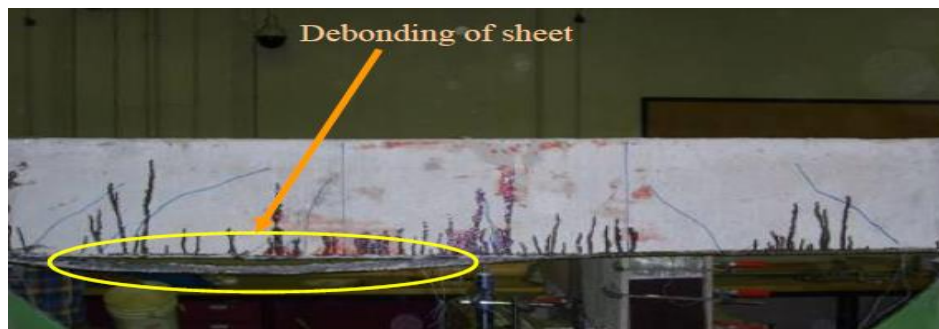


Figure 19: Experimental structural crack pattern for strengthening RC beam, (Nadeem, 2009)

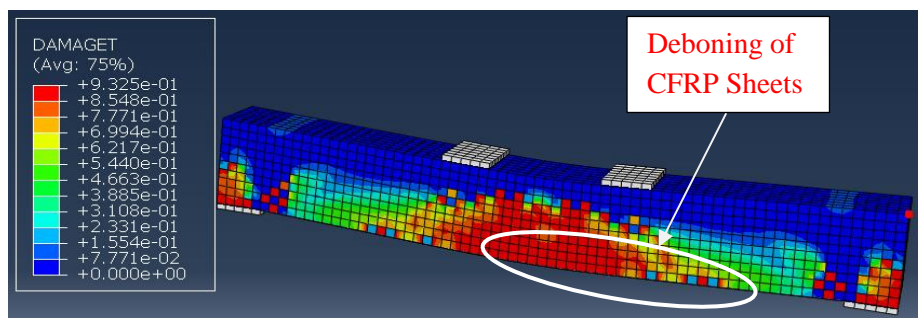


Figure 20: Structural crack pattern for strengthening RC beam, group-1 (FE Analysis)

The failure mode observed from experiments was an intermediate crack induced debonding; where the failure propagated horizontally towards the CFRP sheets end as seen in figure above. This is in agreement with the failure mode reported from finite element analysis reported in Figure 20.

From the above discussion, a comparison of cracking pattern between finite elements prediction and experimental observations was also carried out for the remaining of the RC beams strengthened with CFRP sheets. This comparison is shown in Figure 21-24.

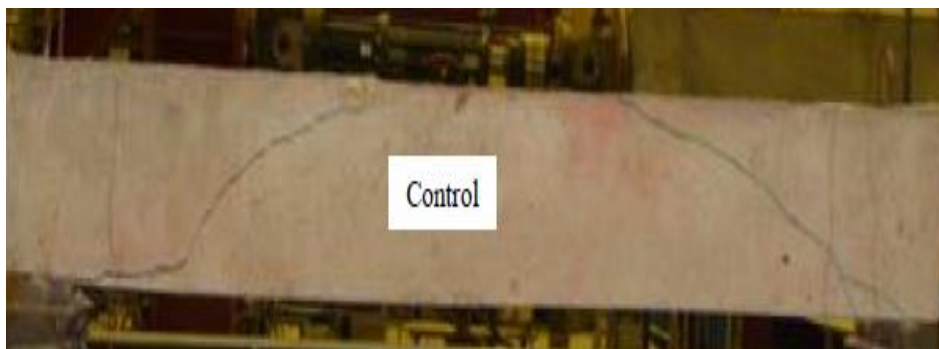


Figure 21: Crack pattern for control beam, group-2_ (Nadeem A.S 2009). (Experiments)

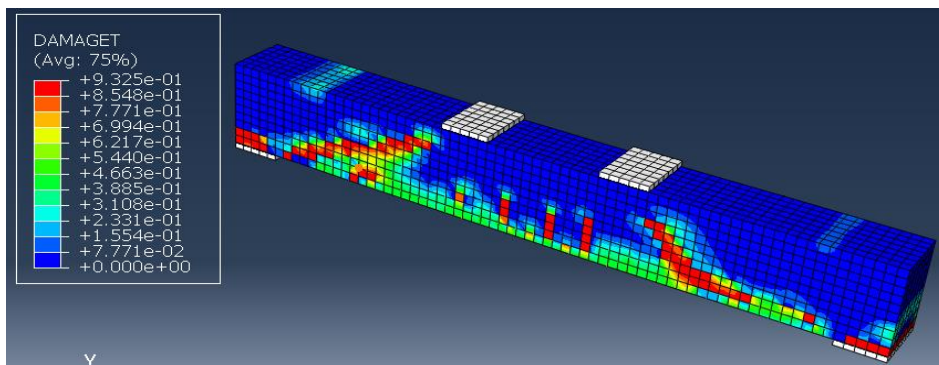


Figure 22: Crack pattern for control beam, group-2. (FE Analysis).



Figure 23: Crack pattern for strengthening RC beam, group-2_ (Nadeem A.S 2009). (Experiments)

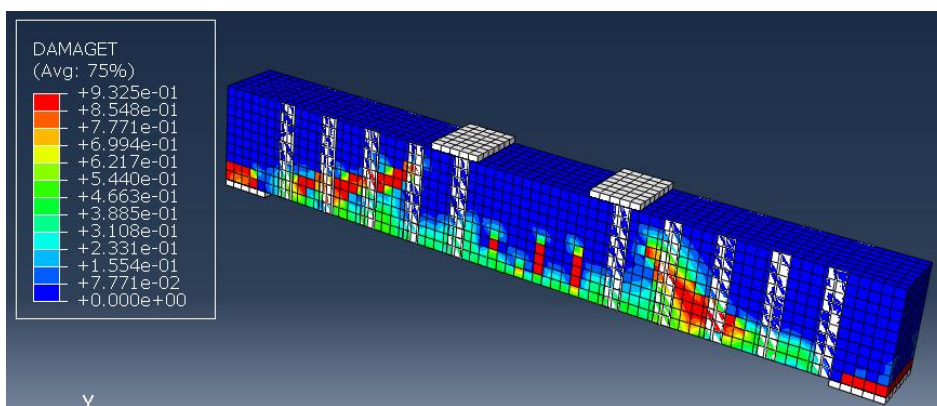


Figure 24: Crack pattern for strengthening RC beam, group-2. (FE Analysis)

5 Summary of Results

The influences of strengthening with CFRP on the ultimate strength of the beams were investigated. It was found that the retrofitted beams model (For flexure or shear) could carry more load than the non-retrofired beams and it has been shown that the use of CFRP can significantly increase the ductility as well as the ultimate strengths of reinforced concrete beams.

The adopted material constitutive models, i.e. a concrete damaged plasticity model for concrete; a linear elastic perfectly plastic model for steel; a linear elastic model for CFRP material are all able to capture the mechanical behavior of the respective materials.

The comparison between experimental and the numerical load–deflection curves for the mid span deflection shows that the finite element load deflection curves are relatively stiffer than the experimental plots. This could be attributed to the nature of the finite element mathematical models which overestimates the stiffness coefficients of structural elements.

Cracking in reinforced concrete beams is initiated when the maximum principal stress exceeds the tensile strength of concrete. The crack patterns from the finite element models correspond well with the observed failure modes of the experimental tests.

6 Conclusions and Future Work

The present study focused on developing a finite element model to investigate the behavior and failure mechanisms of RC beams strengthened with CFRP composites. Structural repair and strengthening are gaining extensive applications around the world. For the development of some recommendations about repair and strengthening, it is necessary to carry out further numerical and experimental investigations for complete understanding of structural behavior of RC elements strengthened with CFRP materials. This section highlights some issues that future research should address for the development of structural strengthening design guidelines.

Based on findings, discussions and conclusions, the following recommendations are made:

- Future researches should address the analytical studies based on the stress analysis and nonlinear material behavior for RC beams strengthened with CFRP composites. These studies in addition to finite elements predictions and experimental observations will help in the development of accurate design guidelines.
- Efforts should be made to simulate the practical situations of damaged RC beams, i.e. in service and the steel reinforcement corroded.
- For those who will be interested in crack pattern for RC beams, force control is the best choice while displacement control is suited for failure mechanisms capturing since it also considers the descending part of the force relations. In addition, concrete smeared cracking model should be applied to RC beams strengthened with CFRP materials and compared with concrete damaged plasticity model in Abaqus. Both models should be compared with experimental findings to assess their effectiveness.

- The bond behavior between both concrete and CFRP material and both longitudinal and transversal steel should be modeled accurately instead of assuming a perfect bond as done in the present study.
- In addition, much effort should be dedicated to the investigation of the behavior of interface between both concrete and CFRP material.
- The effect of parameters should be investigated further with the same type of approach that was used in this research. Build similar finite element models as used in this research and vary only one parameter and run an analysis. Do this for all parameters and study how each parameter affects the strains in the CFRP at failure.

References

- [1]Ahmed, B. N. and Daoud, O. M. (2018). Finite element analysis of reinforced concrete beams strengthened with Carbon fiber reinforced polymer composite sheets. Master of Science in Building Technology Thesis, University of Khartoum, Sudan.
- [2]Jumaat, M.Z., Kabir, M.H. & Obaydullah, M. (2006). A review of the repair of reinforced concrete beams. *Journal of Applied Science Research*. 2 (6): 317-326.
- [3]Nadeem A. Siddiqui, "Experimental investigation of RC beams strengthened with externally bonded FRP composites",6(2009)343-362
- [4]Simulia (2013), Getting Started with Abaqus, Keyword Edition.
- [5]Mercan, B. (2011), "Modeling and behavior of prestressed concrete spandrel beams", Ph.D Thesis, University of Minnesota, Minneapolis and St. Paul, Minnesota.
- [6]Chen, G.M. (2010), "Behaviour and strength of RC beams shear-strengthened with externally bonded FRP reinforcement", Department of Civil and Structural Engineering, the Hong Kong Polytechnic University, Hong Kong, China.
- [7]Yang, Z.J. and Chen, J.F. (2005), "Finite element modelling of multiple cohesive discrete crack propagation in reinforced concrete beams", *Eng. Fract. Mech.*, 72(14), 2280-97.

- [8] Yang, Z.J. and Chen, J.F. (2004), "Fully automatic modelling of cohesive discrete crack propagation in concrete beams using local arc-length methods", *Int. J. Solid. Struct.*, 41(3-4), 801-26.
- [9] Yang, Z.J., Chen, J.F. and Proverbs, D. (2003), "Finite element modelling of concrete cover separation failure in FRP plated RC beams", *Constr. Build. Mater.*, 17(1), 3-13.
- [10] De Borst, R., Remmers, J.J.C., Needleman, A. and Abellan, M.A. (2004), "Discrete vs. smeared crack models for concrete fracture", *Int. J. Numer. Anal. Meth. Geomech.*, 28(7-8), 583-607.
- [11] Lubliner, J., J. Oliver, S. Oller, and E. Onate, "A plastic-damage model for concrete," *Int.J.Solids Structures* 25(3), pp. 299-326, 1989.
- [12] Lee, J., and G. L. Fenves, "Plastic-damage model for cyclic loading of concrete structures," *J.Eng.Mechanics* 124(8), pp. 892-900, 1998.
- [13] CEB-FIP, "Model Code 1990," Thomas Telford, London, 1993.
- [14] Vonk, R. A., "A micromechanical investigation of softening of concrete loaded in compression," *Heron* 38 (3), pp. 3-94, 1993.
- [15] Van Mier, J. G. M., "Strain-softening of concrete under multiaxial loading conditions," PhDthesis, Techn. Univ. Eindhoven, 1984.
- [16] Krätzig, W. B., and R. Pölling," An elasto-plastic damage model for reinforced concrete with minimum number of material parameters," *Computer and Structures* 82, pp. 1201-1215, 2004.
- [17] Sinha, B. P., K. H. Gerstle, and L.G. Tulin, "Stress-strain relations for concrete under cyclic loading," *Journal of the ACI* 61 (2), pp. 195-211, 1964.
- [18] Hordijk, D. A., "Tensile and tensile fatigue behaviour of concrete; experiments, modeling and analyses," *Heron* 37(1), pp. 3-79, 1992.
- [19] Hillerborg, A., "Analysis of one single crack," *Fracture mechanics of concrete*, ed. By Wittmann, and F. H. Elsevier, pp. 223-249, Amsterdam, 1983.
- [20] Bazant, Z. P., and B. H. Oh, "Crack band theory for fracture of concrete," *Matériaux et Constructions* 16 (93), pp. 155-177, 1983.

- [21] Reinhardt, H. W., and H. A. W. Cornelissen, "Post-peak cyclic behaviour of concrete in uniaxial tensile and alternating tensile and compressive loading," *Cement and Concrete Research* 14 (2), pp. 263-270, 1984.
- [22] Ibrahim M. Metwally, 2014. Three- dimensional nonlinear finite element analysis of concrete deep beam reinforced with GFRP bars. Housing and Building National Center. *HBRC Journal* (2017)13,25-38