



## Technical Efficiency of Sugar Industry in Sudan: Stochastic Frontier Approach

Ibrahim Onour

*School of Management Studies, University of Khartoum  
Khartoum, Sudan (E-mail: [onour@uofk.edu](mailto:onour@uofk.edu))*

**Abstract:** This paper employs Stochastic Frontier Analysis (SFA) to estimate technical efficiency of major sugar producers in Sudan, that includes Kenana Sugar Company and four producers of Sudan sugar company (SSC) which include Sennar, Assalaya, New Halfa, and Al-Genied manufacturers. The production function of sugar output employs three inputs: capital, labor, irrigated area. The finding of the paper indicate technical inefficiency (distance from optimum production frontier) of Sudan sugar company is about 8 percent, implying output loss of (5,000) tons of sugar per annum for each producer. Estimation results in the paper also indicate Kenana Sugar Company is performing at the highest level of efficiency in the group with only 0.12 percent inefficiency level. The output loss due to such technical inefficiency for Kenana Sugar Company is estimated 360 tons of sugar per annum. The finding in the paper also indicate a major source of the inefficiency of SSC producers is over staff of employment (decreasing return to scale to labour input) and under utilization of the available capital inputs (increasing return to scale to capital input).

**Keywords:** Stochastic frontier; Sugar industry; Technical efficiency; Sudan

### 1. INTRODUCTION

The efficiency of a manufacturing firm has two components: technical efficiency and allocative efficiency. Technical efficiency (TE) measures the ability of the firm to produce maximal potential output from a given input. Allocative efficiency (AE) measures the ability of the firm to utilize the cost-minimizing input ratios or revenue-maximizing output ratios. One needs to be technically efficient before one can be allocatively efficient and attainment of both is required for economic efficiency [1]-[5]). Studies on efficiency measurement decomposed technical efficiency further into pure technical and scale efficiency. Scale efficiency measures the optimality of the firm's size where average and marginal products are equal, [6]-[10]. Scale inefficiency takes two forms- either increasing or decreasing returns to scale. A firm displaying increasing returns to scale (IRS) is too small for its scale of operation. Unit costs decrease as output increase. In contrast, a firm with decreasing returns to scale (DRS) is too large for the volume of activities that it conducts as a result unit costs increase as output increases.

This paper is motivated by the increasing interest in identifying the inefficiency sizes and sources in operating productive units. In Sudan the size of the inefficiency in Sugar industry is very vital for policy makers in this sector, as it matters how to increase the efficiency of sugar manufacturing

in the country to compete with regional and international competitors. In the empirical research Stochastic Frontier Approach (SFA) and Data Envelopment Analysis (DEA) are the most common analytical tools in assessing efficiency performance of productive units. In the sugar industry case we may consider the inputs number of labors, machines working hours, irrigated land area, whereas the output can be either sugar output, or sugar cane production. The remaining parts of the paper include the following: Section two highlights the stochastic frontier methodology, section three discusses estimation results, and section four concludes the findings of the paper.

### 2. METHODOLOGY

Stochastic frontier models allow for technical inefficiency while acknowledging the random shocks that affect output pattern from time to time. The virtue of stochastic production frontier models is that the impact of a shock on output can be separated from the effect of inefficiency on output level. To explain this point the stochastic production frontier model can be stated as:

$$y_i = f(x_i; \beta_n) + v_i - u_i \\ \text{for } i = 1, 2, \dots, k; \\ n = 0, 1, \dots, (k+1) \quad (1)$$

where  $v_i$  is the two-sided noise component, and  $u_i$  is the non-negative technical inefficiency component of the error term. The noise component  $v_i$  is assumed to be iid and symmetric, distributed independently of  $u_i$ . Thus, the error term  $e_i = v_i - u_i$  is asymmetric, since  $u_i \geq 0$ . Assume  $v_i$  and  $v_i$  are distributed independently of  $x_i$ , estimation of equation by OLS provides consistent estimates of the coefficients  $\beta_n$  but not  $\beta_0$ , since  $E(e_i) = -E(u_i) \leq 0$ . If  $u_i = 0$ , then  $e_i = v_i$  the error term is symmetric, and the data do not support technical inefficiency. However, if  $u_i > 0$ , then  $e_i = v_i - u_i$  is negatively skewed, and there is evidence of technical inefficiency in the data. This suggests that a test for the presence of technical inefficiency can be based directly on the OLS residuals. To estimate the technical efficiency of each producer distributional assumption of the error term is the Normal Half-Normal model which is based on the following assumptions:

- (i)  $v_i \sim iid N(0, \sigma_v^2)$
- (ii)  $u_i \sim iid N^+(0, \sigma_u^2)$  that is nonnegative half - normal
- (iii)  $v_i$  and  $u_i$  are distributed independently of each other, and of the regressors.

The density functions of  $u > 0$  and  $v$  are given by:

$$f(u) = \frac{2}{\sigma\sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right)$$

$$f(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{v^2}{2\sigma_v^2}\right)$$

Given the independence assumption, the joint density function of  $u$  and  $v$  is:

$$f(u, v) = \frac{2}{2\pi\sigma_u\sigma_v} \exp\left(-\frac{u^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2}\right)$$

Since  $e = v - u$ , the joint density function for  $u$  and  $e$  is:

$$f(u, e) = \frac{2}{2\pi\sigma_u\sigma_v} \exp\left(-\frac{u^2}{2\sigma_u^2} - \frac{(e+u)^2}{2\sigma_v^2}\right)$$

The marginal density function of  $e$  is obtained by integrating  $u$  out of  $f(u, e)$ , which yields:

$$f(e) = \int_0^\infty f(u, e) du$$

$$= \frac{2}{\sigma\sqrt{2\pi}} \left[ 1 - \varphi\left(\frac{e\lambda}{\sigma}\right) \right] \exp\left(-\frac{e^2}{2\sigma^2}\right)$$

$$= \frac{2}{\sigma} \varphi\left(\frac{e}{\sigma}\right) \phi\left(-\frac{e\lambda}{\sigma}\right)$$

$$\text{where } \sigma = (\sigma_u^2 + \sigma_v^2)^{1/2}, \quad \lambda = \frac{\sigma_u}{\sigma_v}$$

And  $\phi(\cdot)$ , and  $\varphi(\cdot)$  are the standard normal cumulative distribution and density functions respectively. Thus, the normal half-normal distribution contains two parameters,  $\sigma_u$  and  $\sigma_v$ .

The next step is to obtain estimates of the technical efficiency of each producer. This requires extracting the information that

$e_i$  contains on  $u_i$ . This can be obtained from the conditional distribution of  $u_i$  given  $e_i$ , which contains whatever information  $e_i$  contains on  $u_i$ . Jondrow *et al.* [5], [6], [7] indicated that if

$u_i \sim N^+(0, \sigma_u^2)$  the conditional distribution of  $u$  given  $e$  is:

$$f(u|e) = f(u, e)/f(e) = \frac{1}{\sigma_*\sqrt{2\pi}} \exp\left[-\frac{(u-u_*)^2}{2\sigma_*^2}\right] / [1 - \varphi\left(-\frac{u_*}{\sigma_x}\right)]$$

$$\text{where } u_* = -e \frac{\sigma_u^2}{\sigma^2} \quad \text{and} \quad \sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma^2$$

Since  $f(u|e)$  is distributed as  $N^+(u_*, \sigma_*^2)$  either the mean or the mode of this distribution can serve as a point estimator for  $u_i$ . They are given by

$$E(u_i|e_i) = u_{*i} + \sigma_* \left[ \frac{\varphi\left(-\frac{u_{*i}}{\sigma_*}\right)}{1 - \varphi\left(-\frac{u_{*i}}{\sigma_*}\right)} \right]$$

and

$$m(u_i|e_i) = \begin{cases} -e_i \left( \frac{\sigma_u^2}{\sigma^2} \right) & \text{if } e_i \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

Once point estimates of  $u_i$  are made estimates of technical efficiency of each producer can be obtained by  $TE_i = \exp(-\hat{u}_i)$  where  $\hat{u}_i$  is either  $E(u_i|e_i)$  or  $m(u_i|e_i)$ . Empirical estimates of  $\sigma_u$  and  $\sigma_v$  entails that the distribution assumptions of the half -Normal based on the moment equations:

$$V(e) = V(v) + V(u).$$

Skewness ( $e$ ) = skewness ( $u$ ) since  $v$  is symmetric. The left-hand side can be consistently estimated by OLS results:

$$m_2 = \left(\frac{1}{n}\right) \sum e_i^2, \quad m_3 = \left(\frac{1}{n}\right) \sum e_i^3$$

Both functions on the RHS are known for the half-normal and exponential models. In particular, for the half-normal model the moment equations are:

$$m_2 = \sigma_v^2 + [1 - \frac{2}{\pi}] \sigma_u^2$$

$$m_3 = (2/\pi)^{1/2} [1 - \frac{4}{\pi}] \sigma_u^3$$

The solutions are:

$$\hat{\sigma}_u = \left( \frac{m_3 \sqrt{\pi/2}}{1 - 4/\pi} \right)^{1/3} \quad (2)$$

$$\hat{\sigma}_v = m_2 - (1 - \frac{2}{\pi}) \hat{\sigma}_u^2$$

Equation (2) indicates that there is no solution for  $\sigma_u$  if  $m_3$  is not negative.

### 3. RESULTS AND DISCUSSION

To estimate technical efficiency from Equation (1), the functional form can be stated as flexible functional form:

$$\log(y_i) = \beta_0 + \sum_{i=1}^k \beta_i \log(x_i) + e_i \quad (3)$$

where  $e_i = v_i - u_i$

$y_i$  is the processed sugar output, and  $x_i$  is the input which include capital (machine working hours), number of labors, and sugar cane cultivation area.

The translog form in Equation (3) is the most commonly used flexible functional form for production functions. Table (1) report results of technical inefficiency and corresponding output loss (cost of inefficiency) due to mismanagement of production inputs.

Estimation results of OLS coefficients, and frontier coefficients of production inputs, capital, labor, and irrigation area for each producer are reported in Tables (2) – (5). Thus, the three coefficients,  $\beta_1, \beta_2$ , and  $\beta_3$  reflect the impact of capital, labor, and irrigation area respectively on sugar production of each producer. As expected the frontier coefficients are less than the OLS coefficients for all producers.

A negative coefficient implies a decreasing return to scale, whereas a positive one is increasing return to scale. For Assalya, New Halfa, and Kenana, labor productivity is negatively associated with output level, indicating that a major source of technical inefficiency is significantly declining labor productivity. Productivity of labor is positively associated with the output level for Sennar and Al-Genied, though statistically insignificant in the case of the latter producer. The impact of capital input is positively associated with the output level, even though statistically insignificant for the four producers of Sudan Sugar Company. It is only significant for Kenana producer. This implies that also another source of inefficiency for Assalya, Sennar, Al-Genied, and New Halfa is sub-optimal uses of the two major inputs: capital and labor.

#### 4. CONCLUSIONS

This paper applies stochastic frontier estimation technique under the assumption of half-normal distribution for error terms. Estimation results indicate annual average output loss due to technical inefficiency for each of the four government sugar producers: Sennar, Assalya, New Halfa, and Al-Genied is estimated at 5,000 tons of sugar per annum, whereas for Kenana company output loss due to 0.12 percent inefficiency level is approximated at 360 tons of sugar per annum. It is also indicated that for Assalya, New Halfa, and Kenana, labor productivity is negatively associated with output level, revealing that a major source of technical inefficiency for these three producers is decreasing return to scale for the labor input. However, for Sennar and Al-Genied labor productivity is positively associated with the output level, though statistically insignificant in the case of the latter producer. The impact of capital input is positively associated with the output level, for the four producers of Sudan Sugar Company and Kenana as well. This result indicates that a source of inefficiency for Sudan Sugar Company producers is mismanagement of the two major production inputs, capital and labor.

**Table 1.** Technical inefficiency and output loss

Company	Technical inefficiency (%)	Average annual output loss (Tons of sugar)*
Assalya	8.90	5079
Sennar	6.20	3830
AL-Genied	11.00	6686
New Halfa	7.60	4474
Average SSC**	8.40	5017
Kenana	0.12	360

\*Due to technical inefficiency.

\*\*SSC=Sudan Sugar Company.

**Table 2.** Assalya OLS and frontier coefficients values

parameters	OLS Coefficient	P-value	Frontier coefficient
Constant	-7.030	0.30	-7.390
$\beta_1$	0.096	0.67	0.007
$\beta_2$	-0.026	0.96	-0.115
$\beta_3$	1.760	0.00	1.671
$R^2$	0.670	-	-
LLF	2.33	-	-

**Table 3.** Sennar OLS and frontier coefficients values

parameters	OLS Coefficient	P-value	Frontier coefficient
Constant	2.20	0.53	2.130
$\beta_1$	0.27	0.24	0.208
$\beta_2$	0.95	0.00	0.888
$\beta_3$	0.09	0.61	0.028
$R^2$	0.47	-	-
LLF	14.50	-	-

**Table 4.** Al-Genied OLS and frontier coefficients values

parameters	OLS Coefficient	P-value	Frontier coefficient
Constant	-16.92	0.00	-16.81
$\beta_1$	0.47	0.18	0.36
$\beta_2$	0.15	0.57	0.04
$\beta_3$	2.29	0.00	2.18
$R^2$	0.74	-	-
LLF	-2.84	-	-

**Table 5.** New Halfa OLS and frontier coefficients values

Parameters	OLS Coefficient	P-value	Frontier coefficient
Constant	2.310	0.00	2.240
$\beta_1$	0.084	0.63	0.008
$\beta_2$	-0.890	0.03	-0.960
$\beta_3$	1.800	0.00	1.730
$R^2$	0.980	-	-
LLF	10.300	-	-

**Table 6.** Kenana OLS and frontier coefficients values

Parameters	OLS Coefficient	P- value	Frontier coefficient
Constant	6.76	0.00	6.75
$\beta_1$	1.17	0.00	1.16
$\beta_2$	-0.17	0.00	-0.17
$\beta_3$	-	-	-
$R^2$	0.98	-	-
LLF	17.5	-	-

## REFERENCES

- [1] Coelli, T. (1996), "A guide to FRONTIER version 4.1: a computer program for stochastic frontier production and cost function estimation". *CEPA (Centre for Efficiency and Productivity Analysis) Working Paper 96/07*, Department of Econometrics, University of New England, Australia.
- [2] Coelli, T.J. (1995), "Estimators and hypothesis tests for a stochastic frontier function: a Monte Carlo Analysis", *Journal of Productivity Analysis* **6**: 247–268.
- [3] Jondrow J., Lovell C., Materos I., 1982, "On the estimation of technical inefficiency in the stochastic frontier production function model", *Journal of Econometrics* 19: pp 233-238.
- [4] Battese, G.E. and Coelli, T.J. (1988), "Prediction of technical efficiencies with a generalized frontier production and panel data", *Journal of Econometrics* **38**: 387–399.
- [5] Aigner, D., Lovell C., and Schmidt P., 1977, "Formulation and estimation of stochastic frontier production function models" *Journal of Econometrics*, vol 6(1), pp21-37.
- [6] Miller S, and Noulas A., (1996) "The Technical Efficiency of Large Bank Production, *Journal of Banking and Finance*, 20, pp. 495 – 509.
- [7] Rezvanian R.; and Mehadian S., (2002) "An Examination of Cost Structure and Production Performance of Commercial Banks in Singapore" *Journal of Banking and Finance*, 26, pp.78-98.
- [8] Subal C., and Lovell C., 2003, "Stochastic Frontier Analysis", Cambridge University Press.
- [9] Green W., 1997, "Frontier Production Functions" in M.H. Pesaran, and P. Schmidt, eds, *Handbook of Applied Econometrics*, Volume 2: Microeconomics, Oxford: Blackwell Publishers, Ltd.
- [10] Meeusen, W. and Broeck, J. (1977), "Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error". *Int. Econ. Rev.* **18**, 435-44.