Excel as a Modelling Platform for Thermodynamic Optimisation Analyses

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Abstract: Property tables are not suitable for sensitivity and optimisation analysis of complex thermodynamic systems that require the thermodynamic properties to be repetitively determinated at a large number of points in the system. This paper shows how Microsoft Excel can be used to develop computerised tables that automate the evaluation of thermodynamic properties. The property tables were copied in Excel and Visual Basic for Applications (VBA) was used to develop a number of user-defined functions that automate the interpolation of thermodynamic data. While maintaining the use of thermodynamic tables, which is an important educational process, the computerised tables can be a useful teaching aid that helps students to have better understanding of thermodynamics. The usefulness of the computerised tables is demonstrated by analysing the gas-turbine cycle with intercooling reheating and regeneration. Using the exact variable-specific heat method, the cycle was analysed for the effects of the inter-stage pressures on the plant's thermal efficiency and net specific-work. The paper shows that more accurate estimates of the optimum values of these pressures could be obtained than usually given by the widely adopted approximate constant-specific heat method. The Excel-based tables are particularly useful in developing countries like Sudan, where dedicated educational software is lacking.

Keywords: DC motor; Thermodynamics; Computerised tables; Modelling platform; Excel; VBA.

1. INTRODUCTION

Thermodynamics is a fundamental subject for all engineering disciplines that deal with energy conversion systems and their impacts on the environment. Understanding the laws and principles of thermodynamics helps engineers to design energy systems that utilise the limited energy resources efficiently while minimising their impact on the environment. To give engineering students deeper insight in the principles of thermodynamics and their practical applications, most textbooks now include problems that require the students to perform sensitivity and optimisation analyses. Such an approach is particularly useful in teaching thermodynamics; which is generally perceived by students as a difficult and boring subject [1-3]. However, in the case of thermodynamic, sensitivity and optimisation analyses require repetitive evaluation of the thermodynamic properties at different states of the system; which is difficult to perform using property tables. Therefore, thermodynamic textbooks require the use of suitable modelling software such as Interactive Thermodynamics (IT) and Engineering Equation Solver (EES) [4-6]. Since these proprietary applications are not available to educational institutions in many developing countries like Sudan, students have less chance to enhance their understanding of the subject or make use of its principles to solve practical problems later.

A modelling platform that is available to many engineering colleges worldwide is Microsoft Excel [7,8]. With its flexible statistical and charting tools, Excel has become the mostly used application for data analysis and presentation in engineering and non-engineering disciplines. However, the capability of Excel is not limited to these two functions as the software developers have equipped it with special tools that greatly enhance its computational capability and allow its user to perform more demanding calculations, such as iterative solutions of non-linear equations and optimisation. The computational capabilities of Excel as a modelling platform can be extended further by taking advantage of Visual Basic for Applications (VBA), which is also packaged freely with every PC, to develop special user-defined functions (UDFs). Since Excel itself is not equipped with built-in function for calculating thermodynamic properties, it has to be linked with dedicated property software, such as REFPROP [9], in order to enable its users to perform thermodynamic analyses. REFPROP enables accurate determination of the thermodynamic properties for numerous fluids by providing appropriate property functions. However, apart from its

proprietary rights, the software is more designed for advanced Excel users than for educational purposes where the use of thermodynamic tables is part of the learning process.

Realising the potential of Excel as a teaching aid, a number of educational institutes developed their own add-ins that enable students to use the spreadsheet for performing thermodynamic analyses. An example of such efforts is that of the research team at the University of Alabama [12,13] who developed a set of computational and organisational tools to be used in the thermodynamic and heat transfer courses. Using their VBA-developed custom functions, it is possible to evaluate the properties from given values of two other intensive properties. For example, the function h_pT(p_1;T_1;"R22","SI") determines the enthalpy of R22 from known values of pressure (p_1) and temperature (T_1) in SI units. The library also includes R134a, water and ideal gases. However, like REFPROP, property functions are used instead of tables. Table interpolation is seen by many as an important educational process not for thermodynamics alone but for engineering analyses in general [1]. Therefore, thermodynamics tables should not be replaced by property software at least for teaching basic thermodynamics courses.

The present paper presents an approach for developing fluid-independent interpolation functions with VBA that allow the thermodynamic tables to be integrated in Excel. Thus, the developed property functions use these interpolation functions rather than mathematical formulae. Apart from preserving the use of thermodynamic tables for educational purposes, the approach minimises the number of custom functions that need to be developed while allowing reversed calculations using the same functions. The paper highlights the capabilities of Excel with these functions by analysing the gas-turbine cycle with intercooling, reheating and regeneration. Since this cycle requires the fluid properties to be evaluated at a large number of points, standard thermodynamic textbooks usually determine the optimum inter-stage pressures between the compressors and turbines by using the approximate method which leads to the relationship $P_i = \sqrt{P_1 P_2}$. Using Excel with the exact variable specific-heat method, the paper shows that more accurate determination of the optimum inter-stage pressures can be obtained.

2. THE GAS TURBINE WITH INTERCOOLING, REHEATING AND REGENERATION

The gas-turbine with intercooling, reheating and regeneration (IRR) shown in Fig. 1 is a good example for demonstrating the concept of thermodynamic optimisation. Analysis of the IRR gas turbine cycle, which involves the evaluation of properties at ten different points, is a tedious task for hand calculation especially when the irreversibility in the compressor and turbine stages and pressure losses are taken into consideration. Therefore, many standard textbooks adopt the "approximate" constant-specific-heat method in order to determine the temperatures and enthalpy differences across each process in the cycle [4,5].

The cycle, which incorporates three modifications to the simple gas turbine cycle; viz. intercooling, reheating, and regeneration, is a very useful example for introducing the
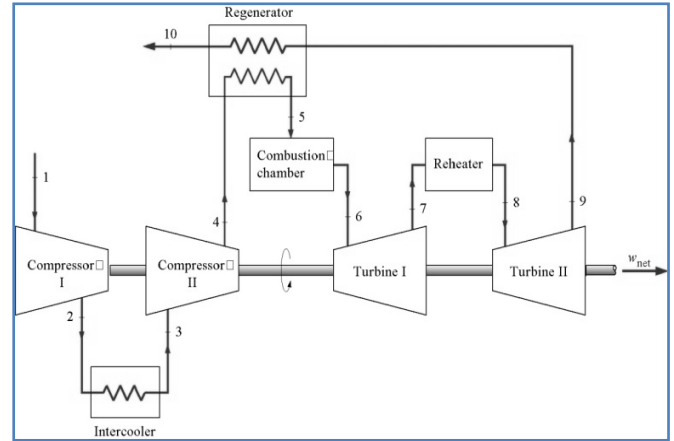

Fig. 1. Schematic of a gas-turbine with intercooling, regeneration, and reheat [5]

concepts of thermodynamic optimisation to the students. Although each of these modifications is meant to improve the gas turbine performance either by increasing the net work output of the gas turbine or its thermal efficiency, these measures can be counter-productive if implemented without proper considerations. For example, regeneration is applied so as to improve the thermal efficiency of the turbine by making use of the exhaust-gas energy. However, regeneration can have an adverse effect if the pressure ratio is higher than a certain value that depends on the temperature ratio (T3/T1) [5]. As explained in standard textbooks, intercooling and reheating which increase the turbine net work output can also reduce the thermal efficiency if applied separately [4]. The inter-stage pressures between the compressors and turbines have specific values that maximise the cycle's thermal efficiency or net work and cannot be chosen randomly. These values can only be determined exactly by performing a sensitivity and optimisation analysis. Although both the net work and thermal efficiency increase by increasing the number of intercooling and reheating stages in the plant, economical considerations put a limit to this number. The same thing can be said about increasing the generator's effectiveness by increasing its heat transfer area. Thus, the problem does not only demonstrate the use of optimisation techniques in designing thermodynamic systems in particular, but the application of thermo-economic optimisation to engineering systems in general.

3. EXACT ANALYSIS OF THE IRR GAS TURBINE CYCLE

Unlike the approximate method, the "exact" method takes into consideration the variation of specific heats with temperature [4]. Using the numbering scheme shown in Fig. 1 to indicate the state of the working fluid entering and leaving each component of the plant, and starting with the known state at the inlet of compressor I, the relative pressure after the first-stage compression (Pr_{2s}) is determined from:

$$Pr_{2s} = Pr_1 \frac{P_2}{P_1} \qquad (1)$$

Pr2s is then used to find T2s, and subsequently h2s, by interpolation from the ideal-gas property tables. Taking into

consideration the adiabatic efficiency of the compressor (η_c), the actual enthalpy at point 2 (h2) is then given by:

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c \qquad (2)$$

Enthalpy of air leaving the second-stage compressor (h₄) and enthalpies of the working fluid leaving the first and second stages of the turbine (h₇ and h₉) are obtained following a similar procedure. The enthalpy of the working fluid leaving the generator is obtained by applying energy balance across the regenerator which leads to:

$$h_5 = h_4 + \varepsilon(h_9 - h_4) \qquad (3)$$

where, ε is the regenerator effectiveness. The heat added, net work output of the turbine, and thermal efficiency of the cycle are then given by:

$$Q_{in} = (h_6 - h_5) + (h_8 - h_7) \qquad (4)$$

$$w_c = (h_2 - h_1) + (h_4 - h_3) \qquad (5)$$

$$w_t = (h_7 - h_6) + (h_9 - h_8) \qquad (6)$$

$$\eta = \frac{w_{net}}{Q_{in}} = \frac{w_t - w_c}{Q_{in}} \qquad (7)$$

The cycle analysis is straightforward in the sense that it does not require an iterative solution or the solution of linear or non-linear systems of equations. However, it is a tedious problem for hand-calculations because it requires the evaluation of thermodynamic properties at a large number of points. Therefore, finding the appropriate values of the cycle parameters that give the optimum performance is a challenging exercise for students even if the approximate method is used [4,5]. The following sections illustrate the usefulness of Excel as a computer-based modelling platform that allows the optimum values of the inter-stage pressures to be determined with the exact method.

4. DEVELOPMENT AND VERIFICATION OF THE COMPUTERISED TABLES

The exact method of solution requires the values of two properties h and Pr at various states of the working fluid. The tabulated values of these two properties for air at selected temperatures within the range 200-1600K were copied into Excel as shown in Fig. 2. To enable Excel to determine values of h and Pr at intermediate temperatures, VBA was used to develop custom property functions for data interpolation. The custom function for the evaluation of enthalpy is given the name "hfT" (i.e. h from T). To find the enthalpy of air at temperature T_1, the function is called as follows:

h_1 =hfT("air";T_1)

The function "hfT" calls another function (named "Linear") that does the linear interpolation. Both functions (i.e. "hfT" and "Linear") are listed in the Appendix. Since the variation of

h is almost linear (Fig. 2), a linear interpolation function was found reasonably accurate even with the large temperature increment, which is 100oC. However, Fig. 2 shows that Pr varies rapidly with temperature and the accuracy of its estimation is bound to be more sensitive to the temperature increment. Therefore, the interpolation function developed for Pr determines the value of Pr from three points instead of two points. The "PrfT" function and the quadratic interpolation function (named "Quadratic") are both listed in the Appendix. The solution also requires the determination of the temperature from a known value of Pr. Therefore, a fifth function was developed for this purpose with the name "TfPr". Like "PrfT", "TfPr" calls the quadratic interpolation function. Note that the two interpolation functions "linear" and "quadratic" are general-purpose functions that can be used for any interpolation operation without modification. Also note that the three property functions "hfT", "TfPr" and "PrfT, which are only slightly different, can be used for different gases by passing the name of the gas to the function. Other required custom functions, such as "Tfh", "sfT", "Tfs", etc., can be developed by a straightforward modification of one of these three property functions.

Before attempting to analyse the effect of the inter-stage pressures on the system performance, the accuracy of the interpolation functions was checked by considering the numerical example given by Moran and Shapiro [4]. In this example, the gas turbine operates at steady state with air entering the compressor at 100 kPa, 300 K. The pressure ratio across the two-stage compressor, and the two-stage turbine, is 10. At the inlets to the turbine stages, the temperature is 1400 K. The temperature at the inlet to the second stage compressor is 300 K. Both the intercooler and the reheater operated at 300 kPa. The isentropic efficiency of each compressor and turbine stage is 80% and the regenerator effectiveness is 80%. Fig. 3 shows the Excel sheet prepared for this example. Note that the sheet shows all the given data and all the calculation steps of the solution.

To make the sheet easy to read, labelled cell references were used, which is an important feature of Excel. Fig. 3 also shows how the equations were inserted in Excel. Table 1 shows the calculated values of the enthalpy at selected points, together with Qin, wc, wt, wnet and η, compared to the corresponding

Table 1. Accuracy of the interpolation functions with different numbers of temperature nodes *N*.

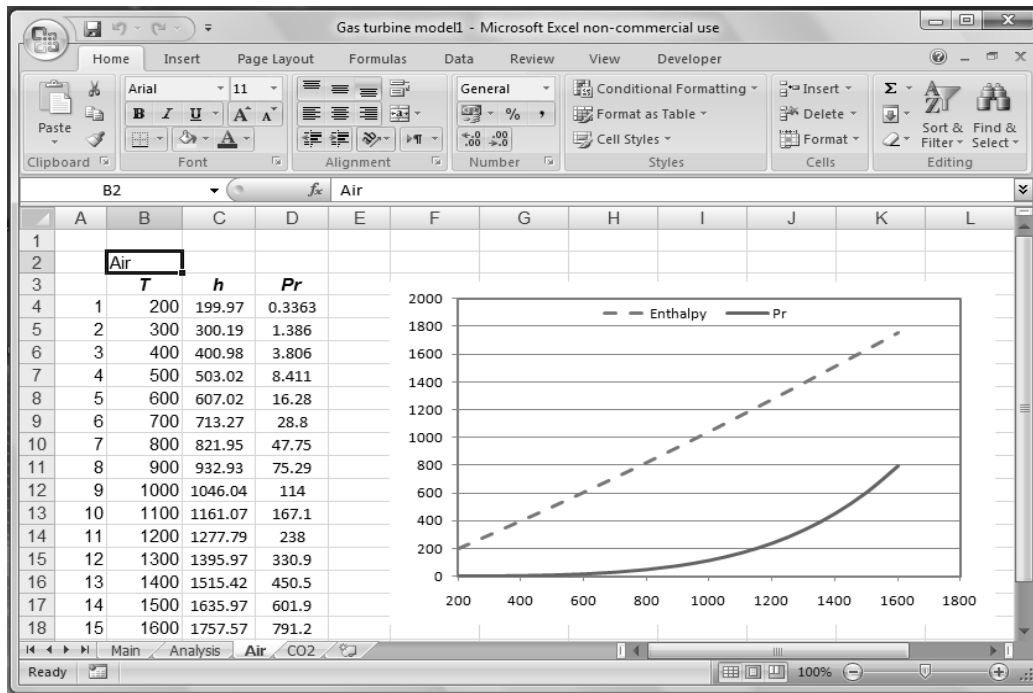|	Excel results			Moran &
	N=15	N=29	N=57	Shapiro [4]
h_2 (kJ/kg)	437.3	438.5	439.2	439.1
h_4 (kJ/kg)	451.6	454.1	454.9	454.7
h_7 (kJ/kg)	1178.9	1179.8	1179.8	1179.8
h_9 (kJ/kg)	1204.7	1205.3	1205.4	1205.2
Q_{in} (kJ/kg)	797.8	796.0	795.7	795.9
w_c (kJ/kg)	288.5	292.3	293.7	293.4
w_t (kJ/kg)	647.2	645.8	645.7	645.8
w_{net} (kJ/kg)	358.7	353.5	351.9	352.4
η	0.450	0.444	0.442	0.443

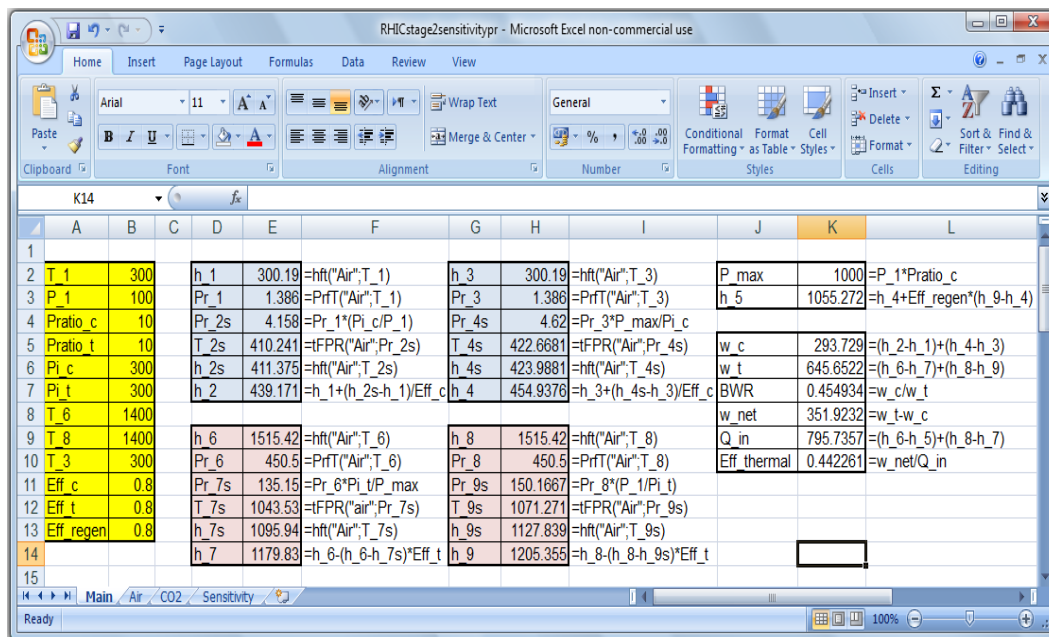**Fig. 2.** Enthalpy and relative pressure data for air

Fig. 3. Sheet 1 (Main) for the gas-turbine problem

values given by Moran and Shapiro [4] who also solved the problem using the variable specific-heat method. In the table, N stands for the number of temperature nodes used in the table, which are 15, 29 and 57. As the figures on the table show, the accuracy steadily improves by increasing N. The values obtained with N=57 are closest to those given by Moran and Shapiro [4].

5. SENSITIVITY AND OPTIMISATION ANALYSIS OF THE IRR GAS TURBINE CYCLE

An important advantage of the computerised table and its associated interpolation functions is their ability to help students perform sensitivity and optmisation analyses. While sensitivity analyses are time-consuming with hand-calculators, cycle optimisation is not possible without computer-based solution methods. Therefore, traditional teaching methods use

the constant specific-heat method to determine the "optimum" inter-stage pressures from the well-known relationship: $P_i = \sqrt{P_1 P_2}$ for both intermediate pressures. Thus, for the numerical example considered above, the method gives $P_2 = P_7 = \sqrt{100 \times 1000} = 316$ kPa (Note that this is slightly higher than the value used in the example, which is 300 kPa). Although these pressures may give the maximum net work, they do not lead to the maximum thermal efficiency.

To study the effects of the two itermediate pressures on the gas-turbine performance, the Excel sheet prepared for the cycle with two-stages of compression and expansion was used to calculate the net specific work and thermal efficiency for different values of the two pressures Pi_c and Pi_t. Fig. 4 shows the variation in the thermal efficiency and specific net-work at an intermediate expansion pressure Pit = P7 = P8 of 316 kPa but different values of the intermediate compression pressure Pic = P2 = P3. Fig. 5 shows the same at an intermediate compression pressure of 316 kPa but different values of the intermediate expansion pressure. As the figures show, the maximum efficiency and the maximum specific net-work occur at different values of the intermediate pressures. The specific combinations of Pic and Pit that give the maximum net specific work or the maximum thermal effficiency could be obtained by using Excel's Solver. While the values of the intermediate pressures that maximise the specific net work are the same and equal to 316.3 kPa, these values do not yield the maximum thermal efficiency, which is usually the real concern for gas turbine designers and operators. Although these findings can now be easily demonstrated to students using educational software, this was not the case few years back when computer-based solution methods were less common [14-16].

The computerised table could also be used to study the effect of increasing the number of compression and expansion stages on the cycle performance compared to that of a simple regenerative cycle. Proper modifications were made to the Excel sheet developed for the two-stage gas turbine in order to perform this analysis. For the simple regenerative cycle with a pressure ratio of 10, the specific net work and thermal efficiency were 216.30 kJ/kg and 34.57%, respectively. Excel Solver was used to determine the optimum values of the intermediate pressures for three-, four-, and five-stages of compression and expansion. The corresponding values of specific net work and thermal efficiency were compared with those of the simple regenerative cycle. Fig. 6 shows the percentage increase in the specific net work and thermal efficiency with different stages of intercooling and reheating. The figure shows that the specific net work continues to increase with significant proportions until five stages of compression and expansion but the increase in thermal efficiency rapidly diminishes after three stages.
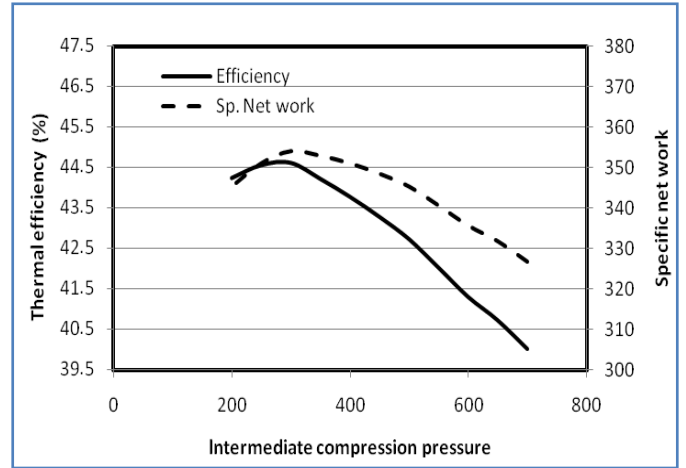

Fig. 4. Variation of the thermal efficiency and specific net work with intermediate compression pressure at Pit = 316 kPa
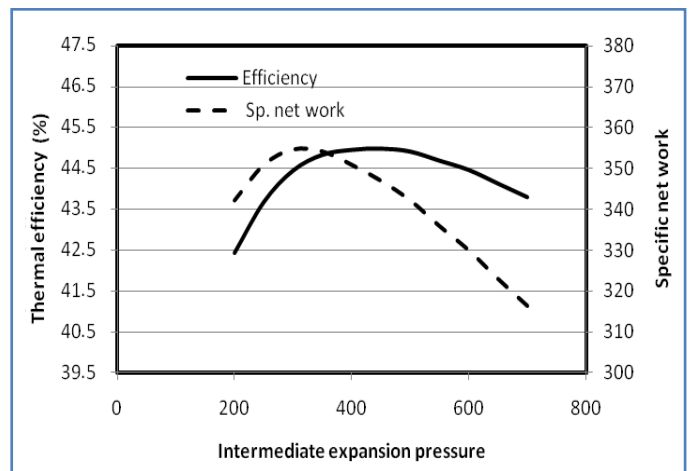

Fig. 5. Variation of the thermal efficiency and specific net work with intermediate expansion pressure at Pic = 316 kPa
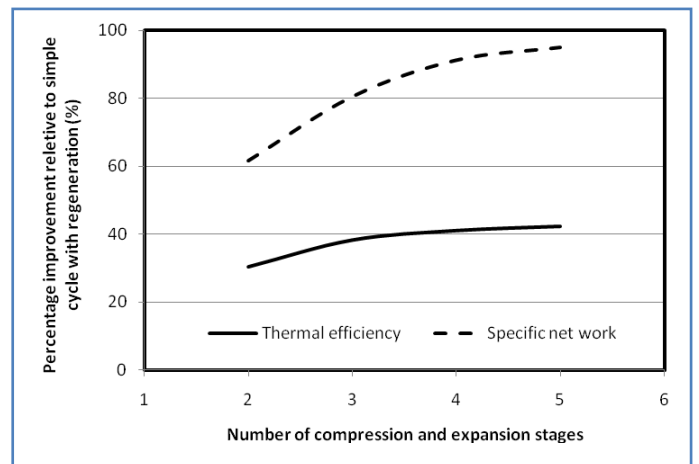

Fig. 6. Percentage increase in thermal efficiency and specific net work compared to those of a simple regenerative cycle

6. CONCLUSIONS

Sensitivity and optimisation analyses, such as the one presented here for the IRR gas-turbine cycle, give the students deeper insight in the principles of thermodynamics and their application in the design process. However, such analyses cannot be performed using conventional proerty tables. The present paper introduced Excel-based thermodynamic tables that enable students to perform such analyses with minimum programming effort. Moreover, using the computerised tables, the analyses can be performed with the exact method of analysis instead of the approximate constant-specific heats method.

Unlike previous attempts to use Excel as a modelling platform for thermodynamic analyses, VBA is used here to develop interpolation functions instead of property functions. The present functions calculate the sought property by interpolation of data which is copied from the thermodynamic tables into Excel. The property functions can also be modified easily to enable the interpolation of any combination of two tabulated properties, e.g. Pr from h., h from Pr, etc. Tables for saturated liquid or saturated vapour phases, where properties can be determined by either the saturation temperature or the saturation pressure, can also be included by the addition of similar property functions. However, the thermodynamic properties of superheated vapours depend on both pressure and temperature and, therefore, the present property functions need suitable modifications in order to allow for such situations.

With its Solver, Excel can also be used to determine optimum parameters in more complex systems than the IRR gas turbine considered in the present analysis. In the present analysis, Excel Solver was used to optimise the thermal efficiency (η) by adjusting the values of the three compressor pressures (PRC1, PRC2 and PRC3). However, Solver can be used to find an optimum solution with up to 200 adjustable cells (e.g. pressures, temperatures, etc.) that are related directly or indirectly to the target cell (e.g. the thermal efficiency). Thus, combined cycles with multiple feed water heaters and reheating, and cogeneration cycles can easily be accommodated.

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APPENDIX

VBA Functions for the Determination of Thermodynamic Properties
Function *hfT*

```
Function hfT(gas$, T)
  With Worksheets(gas$)
    ifirst = 4 'Vertical location of the first cell on the sheet
    jfind = 3 'Horizontal location of the h column on the sheet
    jfrom = 2 'Horizontal location of the T column on the sheet
    hfT = linear(gas$, ifirst, jfind, jfrom, T)
  End With
End Function
```

Function PrfT

```
Function PrfT(gas$, T)
With Worksheets(gas$)
ifirst = 4 'Vertical location of the first cell on the sheet
jfind = 4 'Horizontal location of the Pr column on the sheet
jfrom = 2 'Horizontal location of the T column on the sheet
PrfT = quadratic(gas$, ifirst, jfind, jfrom, T)
End With
End Function
```

Function TfPr

```
Function TfPr (gas$, Pr)
With Worksheets(gas$)
ifirst = 4 'Vertical location of the first cell on the sheet
jfind = 2 'Horizontal location of the T column on the sheet
jfrom = 4 'Horizontal location of the Pr column on the sheet
TfPr = quadratic(gas$, ifirst, jfind, jfrom, Pr)
End With
End Function
```

The Linear Interpolation Function

```
Function linear(gas$, ifirst, jfind, jfrom, x)
With Worksheets(gas$)
itry = 0
i = 1
Do While itry = 0
yl = ifirst + (i - 1)
yh = yl + 1
Xlower = .Cells(yl, jfrom)
Xhigher = .Cells(yh, jfrom)
If (x = Xlower) Then
linear = .Cells(yl, jfind)
itry = 1
Else
If (x = Xhigher) Then
linear = .Cells(yh, jfind)
itry = 1
Else
If (x > Xlower And x < Xhigher) Then
linear = .Cells(yl,jfind)+(.Cells(yh,jfind) -.Cells(yl, jfind)) _
   *((x-.Cells(yl,jfrom))/(.Cells(yh,jfrom)-.Cells(yl,jfrom)))
itry = 1
End If
End If
End If
i = i + 1
Loop
End With
End Function
```

The Quadratic Interpolation Function

```
Function quadratic(gas$, ifirst, jfind, jfrom, x)
With Worksheets(gas$)
itry = 0
i = 1
Do While itry = 0
yl = ifirst + (i - 1)
y2 = yl + 1
y3 = y2 + 1
xlow = .Cells(yl, jfrom)
xmiddle = .Cells(y2, jfrom)
xhigh = .Cells(y3, jfrom)
If (x > xlow And x < xhigh) Then
delf21 = .Cells(y2, jfind) - .Cells(yl, jfind)
delf32 = .Cells(y3, jfind) - .Cells(y2, jfind)
delx21 = xmiddle - xlow
delx32 = xhigh - xmiddle
delx21sq = xmiddle ^ 2 - xlow ^ 2
delx32sq = xhigh ^ 2 - xmiddle ^ 2
c = (delx32 * delf21 - delx21 * delf32)/(delx32 * delx21sq_
    - delx21 * delx32sq)
b = (delx32sq * delf21 - delx21sq * delf32) / (delx32sq*_
    delx21 - delx21sq * delx32)
a = .Cells(yl, jfind) - b * xlow - c * xlow ^ 2
quadratic = a + b * x + c * x ^ 2
itry = 1
End If
If (x = xlow) Then
quadratic = .Cells(yl, jfind)
itry = 1
End If
If (x = xmiddle) Then
quadratic = .Cells(y2, jfind)
itry = 1
End If
If (x = xhigh) Then
quadratic = .Cells(y3, jfind)
itry = 1
End If
i = i + 1
Loop
End With
End Function
```