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# Analytical Solution of Conservation Equations Governing Desiccant Wheel

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**Abstract**: The objective of this work is to provide analytical solution of the conservation equations governing the desiccant wheel. The conservation equations describing the moisture exchange between wheel matrix and airflows are complicated partial differential equations (PDEs). This complication is brought about by space and time variations of moisture content. In this work conservation equations of moisture in the matrix and in airflow were solved using the method of successive transformation of variables. In this process the complicated PDEs were reduced to an ordinary Bessel differential Eq. of the type $xf'' + f' - xf = 0$; which has a general solution of $f(x) = C_1 I_0(x) + C_2 K_0(x)$. The analytical solution has facilitated exact determination of moisture distribution in the matrix and in supply and regeneration airflows. It can also be used to accurately predict the wheel performance parameters such as moisture removal and latent effectiveness. In addition provision of analytical solution to the problem has made significant contribution to the understanding of the complicated desiccant wheel operation principles.

**Keywords:** *Desiccant wheel; Conservation equation; Moisture distribution*

## 1. INTRODUCTION

Desiccant wheels are included in most heating, ventilation and air reconditioning (HVAC) designs for commercial buildings. The desiccant wheel consists of a circular wheel, called matrix, driven by a motor. The matrix has porous channels. The channels can be of different configurations parallel surfaces, equilateral triangle, square, hexagonal, circular or corrugated geometry [cf. Fig. (1)]. The matrix may be made of metal coated with molecular sieves or silica gel or paper impregnated with lithium chloride. The matrix is usually has a large mass transfer area per unit volume e.g. 4000 $m^2/m^3$ and large number of channels per surface area of the face e.g. 40,000 channels/$m^2$ [1]. The wheel is divided into two parts one for process air and the other for the regeneration air. Water vapor of the process air is adsorbed and stored in the matrix. As the wheel rotates to the regeneration side the adsorbed water vapor is driven off by the hot regeneration air.

Many attempts have been made to predict the desiccant wheel performance parameters' such as moisture removal and latent effectiveness. Semi-empirical, numerical, and analytical approaches have been attempted. Due to similarity between heat and mass transfer, the correlations developed for rotary regenerator are generally used to investigate desiccant wheel. Kays and London correlation for rotary regenerator is widely used to estimate the latent effectiveness of desiccant wheel. Simonson [2] and Simonson *et al.* [3] have further developed Kays and London [4] for latent effectiveness. Despite the simplicity of the differential equations, their solution has been proved to be challenging and performance of energy wheel was widely investigated numerically. A number of numerical solution attempts were reported in the literature. These include the works of Simonson [2], Zheng and Worek [5], Klein *et al*. [6], Casas *et al.* [7] and Sphaier and Worek [8], to mention a few. However, there are a few attempts to make analytical solution of the conservation equations governing desiccant. Rabah and Mohamed [9] have assumed Henry's law of moisture isotherm and time constant of moisture in the gas phase. Their solution produced latent effectiveness correlations with limited range of application.

This work is intended is to develop a method of solution to the conservation equation governing the desiccant wheel. Mass conservation equations for the moisture in the desiccant and vapor in the gas phase will be written for each section of the wheel. The moisture distribution in both the matrix and vapor space will be specifies.

## 2. ANALYTICAL MODEL

The conservation equations for water vapor in the air and the desiccant wheel are written as respectively [2] :

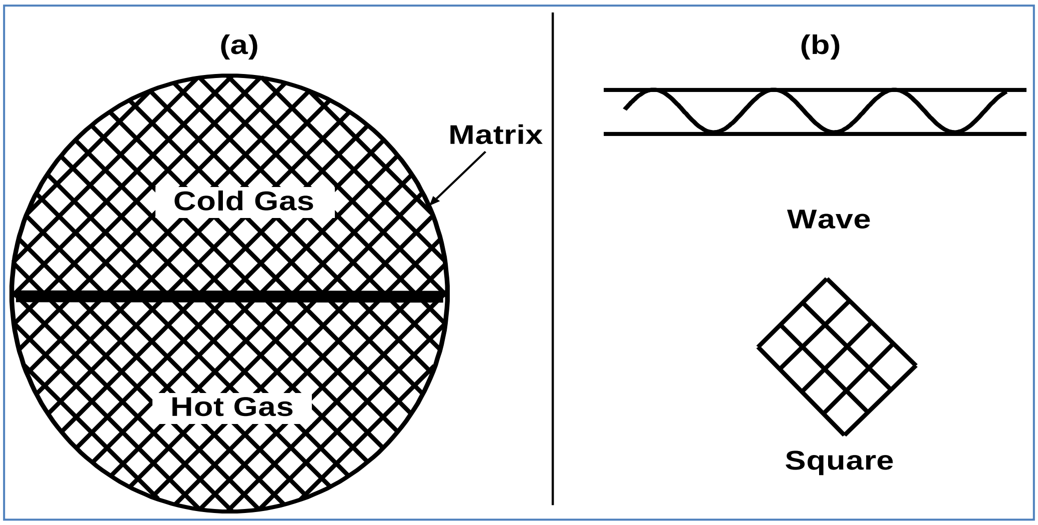


**Fig. 1.** Desiccant wheel configuration; (a) face of entire wheel, (b) tube geometry cross-section

$$A_v \frac{\partial \rho_v}{\partial t} + uA_v \frac{\partial \rho_v}{\partial x} = h_m \frac{A_s}{L}\left(\rho_{vm} - \rho_v\right) \quad (1)$$

$$\rho_d A_d \frac{\partial X_m}{\partial t} = h_m \frac{A_s}{L}\left(\rho_v - \rho_{vm}\right) \quad (2)$$

$h_m$ is the convective mass transfer coefficient (m/s), X is moisture content per desiccant, $A_s$ is the surface area of the desiccant channel and $A_v$ is the channel cross-sectional area (gas flow area), $A_d$ is the desiccant cross-sectional area. u is the air velocity, $\rho_{vm}$ and $\rho_v$ is moisture density at the air and desiccant matrix respectively. $\rho_d$ is dry desiccant density. *t* and *x* are time and space coordinates respectively. The subscript *v* and *m* stand for water vapor and matrix.

The left hand side of Eq. (2) can be transformed using exact derivative as

$$\frac{\partial X_{vm}}{\partial t} = \frac{\partial X_{vm}}{\partial \rho_{vm}} \frac{\partial \rho_{vm}}{\partial t} \quad (3)$$

Hence Eq. (2) becomes

$$\left(\rho_d A_d \frac{\partial X_{vm}}{\partial \rho_{vm}}\right)\frac{\partial \rho_{vm}}{\partial t} = h_m \frac{A_s}{L}\left(\rho_v - \rho_{vm}\right) \quad (4)$$

The next step we seek transformation of space and time coordinates into dimensionless coordinates. Introducing the following dimensionless variables for *t* and *x* as [10],

$$x^* = \frac{x}{L}, \qquad t^* = \frac{1}{t_o}\left(t - \frac{x}{u}\right) \quad (5)$$

where $t_o$ is the period of exposure per cycle for the supply or generation air [s/cycle]. The time and spatial derivatives can be written for any dependent variable, *y*, as follows [2],

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{1}{t_o}\frac{\partial y}{\partial t^*} \quad (6)$$

$$\frac{\partial y}{\partial z} = \frac{\partial y}{\partial x^*}\frac{\partial x^*}{\partial x} + \frac{\partial y}{\partial t^*}\frac{\partial t^*}{\partial x} = \frac{1}{L}\frac{\partial y}{\partial x^*} - \frac{1}{t_o u}\frac{\partial y}{\partial t^*} \quad (7)$$

With these transformation (Eqs 6 and 7) the conservation Eq. (1) becomes

$$\frac{uA_v}{L}\frac{\partial \rho_v}{\partial x^*} = h_m \frac{A_s}{L}\left(\rho_{vm} - \rho_v\right) \quad (8)$$

Similarly Eq. (4) becomes

$$\left(\frac{\rho_d A_d}{t_o}\frac{\partial X_d}{\partial \rho_{vd}}\right)\frac{\partial \rho_{v_d}}{\partial t^*} = h_m \frac{A_s}{L}\left(\rho_v - \rho_{vd}\right) \quad (9)$$

Next we will introduce the dimensionless groups ($NTU$, $Cr_o^*$) and space and time dimensionless parameters ζ and η respectively.

$$NTU = \frac{h_m A_s}{uA_g}, \qquad Cr_o^* = \frac{\rho_d A_d}{A_g u t_o}\frac{\partial X_d}{\partial \rho_{vd}}, \qquad \xi = NTU z^*, \qquad \eta = \frac{NTU}{Cr^*}t^* \quad (10)$$

with these new variables given by Eq. (10), the differential Eqs (8-9) become, respectively,

$$\frac{\partial \rho_v}{\partial \xi} = \rho_{vm} - \rho_v \quad (11)$$

$$\frac{\partial \rho_{vm}}{\partial \eta} = \rho_v - \rho_{vm} \tag{12}$$

Elimination of □$_{vm}$ in Eqs (11-12) yields

$$\frac{\partial^2 \rho_v}{\partial \eta \partial \xi} + \frac{\partial \rho_v}{\partial \eta} + \frac{\partial \rho_v}{\partial \xi} = 0 \tag{13}$$

Eq. (13) is known as Nusselt Eq. in the literature. In fact all analytical solutions of the conservation Eqs ended at Eq. (13). Most of the numerical solution of desiccant wheel are the numerical solution of Eq. (13). The contribution of this works starts from Eq. (13) on.

Now $\rho_v$ will be transformed into a variable *X* as

$$\rho_v = X e^{-\xi - \eta} \tag{14}$$

Insertion of Eq. (14) into Eq. (13) yields Eq. (15)

$$\frac{\partial^2 X}{\partial \eta \partial \xi} - X = 0 \tag{15}$$

Eq. 15 can be solved using Laplace transformation; however, it is a lengthy and complicated procedure. Here we try to introduce a novel route for the solution of (15) by continuing the method of transformation of variables. Because of the chain of transformations, ρ (x, t)→ ρ (ζ,η)→X(ζ,η)→f (β) we called this method "successive transformation of variables". The last ring in this chain is the following transformation of variables

$$X(\xi, \eta) = f(\beta) \tag{16}$$

where

$$\beta = 2\sqrt{\xi \eta} \tag{17}$$

With the transformation given by Eqs (16-17), Eq. (15) is reduced to an ordinary differential Eq. as

$$\beta f'' + f' - \beta f = 0 \tag{18}$$

Eq. (18) can be recognized as a standard form of ordinary Bessel differential Eq., which has the general solution of

$$f(\beta) = C_1 I_o(\beta) + C_2 K_o(\beta) \tag{19}$$

where $I_o$ and $K_o$ are modified Bessel functions of the first and second kind and zero order, respectively, and $C_1$ and $C_2$ are constants. In fact this is an interesting development in the solution of the conservation Eqs of the desiccant wheel. The last transformation step is not only reducing the tedious procedures of using Laplace transformation, namely the inverse Laplace transformation, but it is a simple, clear and straightforward and hence educational step. Transforming Eq. (19) backwards we get

$$X(\xi . \eta) = C_1 I_o\left(2\sqrt{\xi \eta}\right) + C_2 K_o\left(2\sqrt{\xi \eta}\right) \tag{20}$$

The determination of the constant $C_1$ and $C_2$ of Eq. (20) depends on the boundary conditions. For example if the initial boundary condition is ζ = 0 (***x* = 0**) and the density is finite then

$$X(\xi 0 . \eta) = C_1 I_o(0) + C_2 K_o(0) \tag{21}$$

With $K_o(0)=\infty$ , $C_2 = 0$. Hence (21) reduces to

$$X(\xi, \eta) = C_1 I_o\left(2\sqrt{\xi \eta}\right) \tag{22}$$

Again transforming backward to vapor density we get

$$\rho_v = C_1 I_o\left(2\sqrt{\xi \eta}\right) e^{-\xi - \eta} \tag{23}$$

Eq. (23) is an impulse function. Using the principles of convolution, Eq. (23) becomes

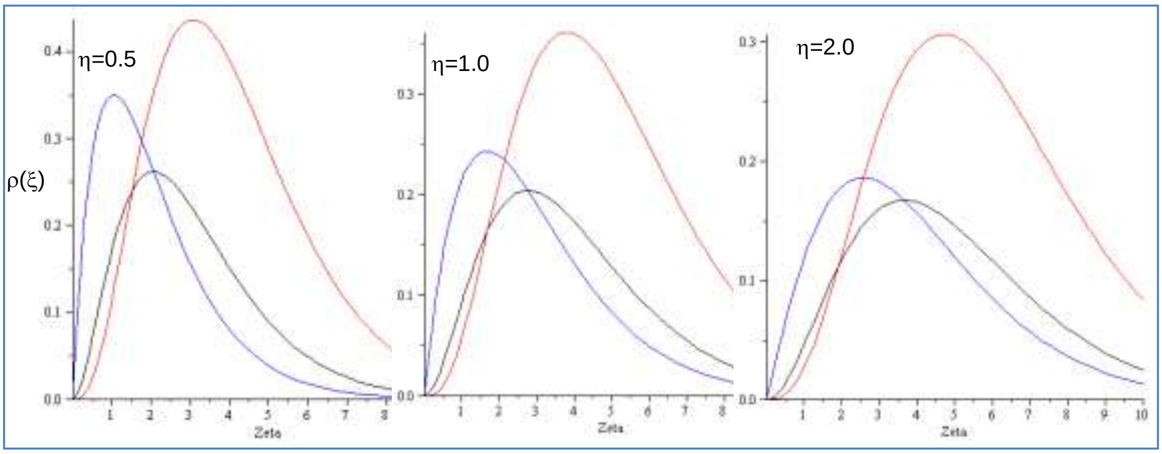


**Fig. 2.** Moisture distribution (blue: Φ=1, black: Φ=x, red: Φ=$x^2$)

$$\rho_v(\xi,\eta) = e^{-\xi-\eta}\int_0^{\xi}\Phi(\lambda)I_o\left[2\sqrt{\eta(\xi-\lambda)}\right]d\lambda \qquad (24)$$

More details on the convolution integral are found in Rabah and Kabelac [11]. The nature of the function Φ depends on the boundary conditions. Knowing the moisture distribution, the wheel performance parameters such moisture removal and latent effectiveness can be determined.

Fig. 2 shows the moisture distribution in the desiccant wheel for wide range of ξ values and Φ function. Experimental data on local measurement of moisture on the wheel is not available and is extremely difficult (if not impossible) to measure. Nevertheless the trend of moisture profile is correctly predicated as attested by the experimental data of Rabah *et al.* [12].

## 3. CONCLUSIONS

Mass conservation equations for the moisture in the desiccant and water vapor in the gas were solved analytical. The solution was straightforward, producing equation for moisture distribution.

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**Nomenclature**

| **Roman** | | **Greek symbols** | | **Superscripts** |
|---|---|---|---|---|
| *A* | Heat transfer area ($m^2$) | *ρ* | Density ($kg/m^3$) | * Normalized |
| $h_m$ | Mass transfer coefficient (m/ s) | Φ | Function (-) | *O* Degree |
| | | *η* | Parameter (-) | |
| L | Length (m) | ξ | Parameter (-) | |
| NTU | Number of transfer units (-) | **Subscripts** | | |
| t | Time (s) | d | Desiccant | |
| u | Velocity (m/s) | m | Matrix | |
| *X* | Moisture in the desiccant (kg/kg) | v | vapor | |