Available online at http://ejournals.uofk.edu

UofKEJ Vol. 1 Issue 2 pp. 1-8 (October 2011)

UNIVERSITY OF KHARTOUM ENGINEERING JOURNAL (UOFKEJ)

Orthogonal Frequency Division Multiplexing Theory and Challenges

Ghassan M. T. Abdalla

Department of Electrical and Electronic Engineering, Faculty of Engineering, University of Khartoum Khartoum, Sudan (Tel.; +249128900087; E-mail: gmtshigidi@yahoo.com)

Abstract: Orthogonal Frequency Division Multiplexing (OFDM) has become the modulation technique used in several present and future communication standards. It is tolerant to frequency selective channels and provides high data rates. In this paper we investigate the theory and challenges of OFDM. A mathematical model is derived and used to explain the principles of OFDM. The model is then used to investigate inter-carrier interference (ICI) in OFDM systems. BER expressions are then derived and used to analyze the performance of OFDM systems with mobility.

Keywords: Doppler Shift; Fading; Inter-Carrier Interference; OFDM.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been adopted as the modulation technique for various current and proposed wireless systems including WiFi, WiMax and digital video broadcast (DVB) [1, 2]. It provides high data rates, high spectral efficiency and dynamic allocation of bandwidth to users using low complexity hardware. It eliminates the need for multi-tap equalizers in frequency selective channels by dividing the available bandwidth into several narrow band channels. These channels can be either allocated to different users (a scheme known as Orthogonal Frequency Division Multiple Access OFDMA) or used solely by a single user to transmit at a high data rate. Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform (FFT) are employed at the transmitter and receiver respectively to modulate and demodulate the signal. Despite its efficiency and high reliability, OFDM faces a number of challenges limiting its performance particularly in mobile links. In this paper we analyze OFDM modulation and investigate the challenges facing it. The rest of the paper is organized as follows: section II introduces the background of OFDM. Section III provides an overview of the operation of OFDM and derives a mathematical model for OFDM systems. The model is used to explain and analyze the challenges facing OFDM in section IV. Section V concludes the paper.

2. BACKGROUND OF OFDM

In wireless communications, the signal transmitted from the source typically experiences attenuation, scattering, reflection and refraction before it reaches the destination. These effects are usually modeled as one or several values known as the channel response [3-6] which is convolved with the transmitted signal. The response of the channel between the transmitter and the receiver is not fixed but varies with time and frequency. The bandwidth upon which the channel response can be assumed fixed (flat) is known as the coherence bandwidth of the channel. If the data is transmitted at high symbol rates, the bandwidth of the signal becomes wide and may exceed the coherence bandwidth of the channel [3, 5]. This distorts the signal and leads to inter symbol interference (ISI). ISI degrades the signal in two ways. First, previously transmitted symbols interfere with the current symbol. Second, part of the current symbol energy is lost as it will cause ISI for subsequent symbols. To eliminate ISI, equalization is usually employed. The equalizer is an adaptive digital filter with a certain number of taps. The weights of the taps in the equalizer are designed such that the combined response of the channel and equalizer is a constant value (flat) within the signal bandwidth. Equalizers suffer from a number of limitations. Finding the optimum weight of each tap is a complicated process which increases exponentially as the length (number of taps) of the filter increases [3, 7]. Moreover, these weights are calculated from a noisy estimate of the channel response and hence the estimation error will be higher compared to the single tap filter needed for flat channels. Another limitation is equalizers are designed with a maximum length (number of taps). Such equalizers will perform poorly if the channel response is longer than the equalizer’s length. To eliminate the need for multi-tap equalizers, it was proposed in the 1960s that the data be split into parallel streams, thus reducing the symbol rate and bandwidth of each stream. If the number of streams is large enough, the bandwidth of each stream can become less than the channel coherence bandwidth and hence each stream experiences a flat channel response. These streams are then modulated using separate orthogonal carriers known as subcarriers. The subcarriers must fit within the bandwidth allocated for transmission but must be far enough so that they do not interfere with each other. The minimum spacing between the subcarriers was found to be 1/T where T is the symbol duration after splitting the data into parallel streams [3, 8].

Fig. 1 shows the spectrum of data modulated using subcarriers with 1/T spacing. The transmission scheme proposed in the 1960s suffered from two limitations. First, a separate modulator is required for each stream hence increasing the cost of the system. Second, any frequency synchronization errors will lead to interference between the subcarriers (see Fig. 1). The risk of interference due to frequency errors can be reduced by increasing the separation between the subcarriers at the expense of bandwidth efficiency [3, 8, 9].

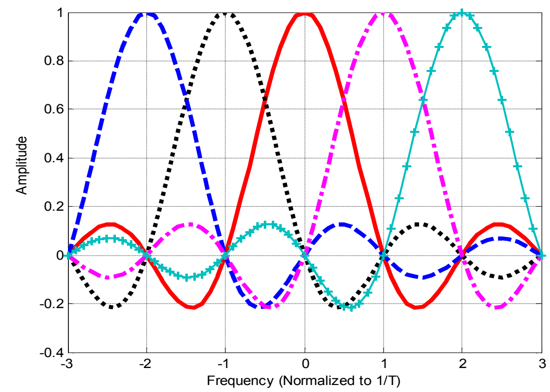

Fig. 1. Illustration of the orthogonality between the subcarriers in OFDM

3. THEORY OF OFDM

The transmitted signal ($s(t)$) using several independent carriers is given by [8, 10, 11]:

$$s(t) = \sum_{p=0}^{N-1} d(p,t) e^{j.(2\pi f_c + p.\Delta\omega)t}$$
$$= e^{j2\pi f_c t} \times \sum_{p=0}^{N-1} d(p,t) e^{j.p.\Delta\omega t} \qquad (1)$$

where $d(p,t)$ is the data for stream p which also represent the subcarrier index, $\Delta\omega$ is the frequency spacing, f_c is the frequency of the first subcarrier, t is the time, N is the number of subcarriers and j is the square root of -1. The receiver samples the received signal, hence, we rewrite Eq. (1) for discrete time ($t = nT$) to get:

$$s(n) = e^{j.2\pi f_c.nT} \times \sum_{p=0}^{N-1} d(p,n) e^{j.p\Delta\omega.nT} \qquad (2)$$

For subcarriers with frequency spacing of $1/NT$, Eq. (2) becomes:

$$s(n) = Ne^{j.2\pi f_c.nT} \times \frac{1}{N} \sum_{p=0}^{N-1} d(p,n) e^{j\frac{2\pi.}{N}p.n} \qquad (3)$$

Considering the summation on the right hand of Eq. (3) we note that this is the Inverse Discrete Fourier Transform (IDFT) with $d(p,n)$ the data at subcarrier p at time sample n. The IDFT can be calculated efficiently using any of the existing Inverse Fast Fourier Transform (IFFT) algorithms. The first factor on the right hand side of Eq. (3) is a multiplication by a carrier that can be achieved by a

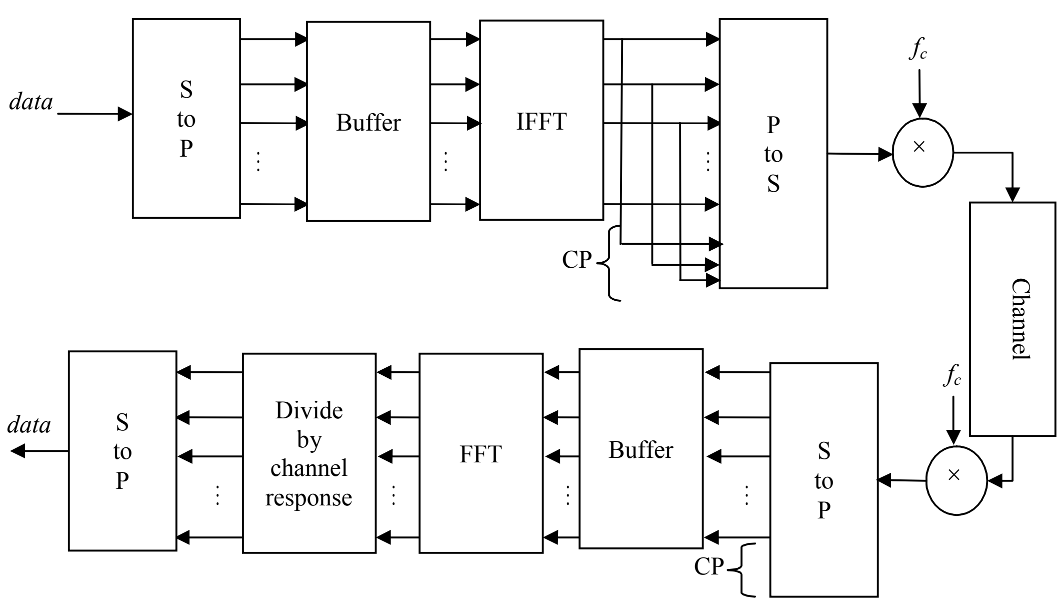

Fig. 2. Block diagram of OFDM transmitter and receiver

modulator. Hence, instead of using N modulators, we can combine an IFFT with a single modulator to achieve the same performance. The receiver first demodulates the signal to remove the first multiplicative factor on the right hand side of Eq. (3), then uses a Fast Fourier Transform algorithm to recover the data ($d(p,n)$) from the summation of Eq. (3). Fig. (2) is a block diagram for the OFDM transmitter and receiver. The role of the cyclic prefix (CP) and division by channel response will be explained later.

Ignoring the CP for now, the transmitted OFDM signal ($s(n)$) consists of the N output values from the IFFT (known as the OFDM symbol) and is transmitted serially with a symbol rate equal to N/T. Hence, the transmitted OFDM signal experiences a frequency selective fading channel as it has N times the bandwidth of individual streams. We prove next that the processing done at the transmitter and receiver, converts this channel into a set of flat fading channels. First, we approximate the channel response with a finite impulse filter (FIR) of length M ($M < N$), and we consider a single transmitted OFDM symbol. The received signal ($r(n)$) after demodulating the carrier frequency (f_c) is given by:

$$r(n) = \sum_{l=0}^{M-1} s(n-l) z_l(n) + w(n) \tag{4}$$

where $z_l(n)$ is the channel response of tap l at time index n and $w(n)$ is the noise sample.

The signal at subcarrier k ($x(k)$) after the FFT processes the received signal ($r(n)$) after demodulating the carrier frequency (f_c) is given by:

$$\begin{aligned} x(k) &= \sum_{n=0}^{N-1} r(n).e^{-j\frac{2\pi}{N}k.n} \\ &= \sum_{n=0}^{N-1}\sum_{l=0}^{M-1} s(n-l) z_l(n).e^{-j\frac{2\pi}{N}k.n} + \sum_{n=0}^{N-1} w(n).e^{-j\frac{2\pi}{N}k.n} \\ &= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{M-1}\left(z_l(n).e^{-j\frac{2\pi}{N}k.n} \sum_{p=0}^{N-1} d(p,n)e^{j\frac{2\pi}{N}p.(n-l)}\right) + \sum_{n=0}^{N-1} w(n).e^{-j\frac{2\pi}{N}k.n} \\ &= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{p=0}^{N-1}\left(d(p,n)e^{j\frac{2\pi}{N}p.n} e^{-j\frac{2\pi}{N}k.n} \sum_{l=0}^{M-1} z_l(n).e^{-j\frac{2\pi}{N}p.l}\right) \\ &\quad + \sum_{n=0}^{N-1} w(n).e^{-j\frac{2\pi}{N}k.n} \end{aligned} \tag{5}$$

Let the N point FFT of the channel response $z_l(n)$ be:

$$\sum_{l=0}^{N-1} z_l(n) e^{-j\frac{2\pi}{N}kl} = h(n,k) \tag{6}$$

$$h(n,p) = \sum_{l=0}^{N-1} z_l(n).e^{-j\frac{2\pi}{N}p.l} = \sum_{l=0}^{M-1} z_l(n).e^{-j\frac{2\pi}{N}p.l} \tag{7}$$

Since it is assumed that the channel response has a maximum length M which is shorter than N, $z_l(n)$ is equal to 0 for $l \geq M$. Substituting (7) in (5), we get:

$$x(k) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{p=0}^{N-1} d(p,n).h(n,p).e^{j\frac{2\pi}{N}n.p}.e^{-j\frac{2\pi}{N}n.k} + W(k) \tag{8}$$

$W(k)$ is the N point FFT of the noise samples. If the channel does not vary within the OFDM symbol we have:

$$h(n,p) = h(p) \tag{9}$$

The data $d(p,n)$ remain fixed for the duration of the OFDM symbol, hence:

$$d(p,n) = d(p,0) = d(p) \tag{10}$$

Substituting (9) and (10) in (8) we get:

$$\begin{aligned} x(k) &= \frac{1}{N}\sum_{p=0}^{N-1} d(p).h(p)\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n.(p-k)} + W(k) \\ &= \sum_{p=0}^{N-1} d(p).h(p).\delta(p-k) + W(k) \end{aligned} \tag{11}$$

$\delta(p-k)$ is the shifted Dirac-Delta function. Therefore Eq. (11) reduces to:

$$x(k) = d(k)h(k) + W(k) \tag{12}$$

Ignoring the noise, dividing $x(k)$ by the channel response $h(k)$ yields the transmitted data.

From the above analysis, we note that the IFFT and FFT algorithms used at the transmitter and receiver eliminated the ISI between the symbols within the OFDM symbol (intra-OFDM ISI). However, if consecutive OFDM symbols are transmitted, ISI between different OFDM symbols occurs (inter-OFDM ISI) and this cannot be resolved.

Fig. 3 illustrates the different types of ISI encountered in OFDM transmission. Fig. 3.a is the sampled channel response of a frequency selective channel. When the symbol shown in the upper part of Fig. 3.b is transmitted through the channel of Fig. 3.a, we receive the signal shown in the lower part of Fig. 3.b which is extended beyond the period of the transmitted symbol. This extension causes ISI between the symbols in a single OFDM symbol (intra-OFDM ISI) as shown in Fig. 3.c. The IFFT-FFT combination resolves this intra-OFDM ISI. Inter-OFDM ISI is illustrated in Fig. (3.d) where the previously transmitted OFDM symbol interferes with the successive OFDM symbol. This ISI degrades the performance of OFDM because the IFFT-FFT combination cannot resolve it. The reason why the OFDM system fails to eliminate inter-OFDM ISI is that the data changes from one OFDM symbol to another, hence the assumption that the data ($d(p,n)$) is independent of n, Eq. (10), is no more valid.

To eliminate inter-OFDM ISI, there are two approaches. The first, known as zero padding, is to insert a gap (zeros) between OFDM symbols. The length of the gap must be at

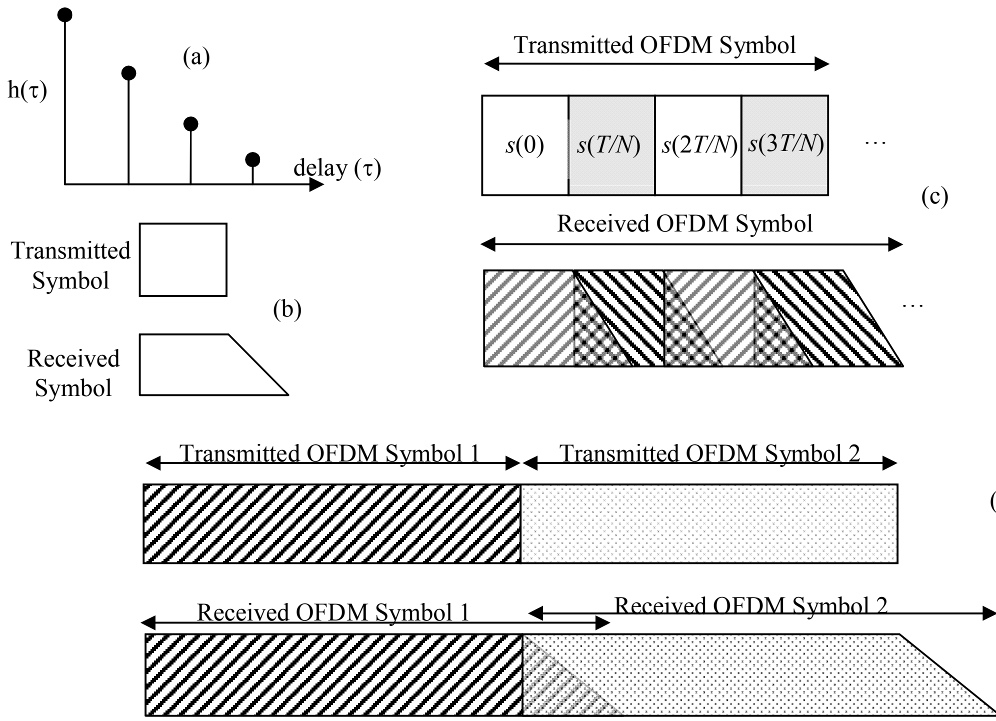

Fig. 3. Different types of ISI encountered in OFDM

least equal to the length of the channel response to avoid the ISI. The second approach is to send the OFDM symbol twice and decode the second OFDM symbol. This way we can ensure that the data is fixed for two OFDM symbols and hence, Eq. (10) remains valid.

The second solution reduces the efficiency of the link to 50%. To improve this, we need to reduce the size of the duplicated transmissions. Note from Fig. 3.d that only the end of an OFDM symbol contributes to the ISI affecting the next symbol. Hence, if only a copy of this part at the end of the OFDM symbol is transmitted before OFDM symbol, we can eliminate the inter-OFDM ISI. This copied data is known as the cyclic prefix (CP). The CP must be at least as long as the channel response. The CP solution is usually preferred to zero padding because it makes the signal periodic from the perspective of the FFT. FFT algorithms require the signal to be periodic to produce accurate results. If the signal is not periodic (as with zero padding), a phenomenon which causes interference between the subcarriers, known as windowing effect, occurs and affects the results. The CP solution, however, consumes more power compared to zero padding. The receiver removes the CP prior to the FFT [1, 8, 10].

4. CHALLENGES FACING OFDM

Beside the usual synchronization, channel estimation and noise issues encountered in most transmission systems, OFDM systems face two unique problems. The first is its high peak to average power ratio and the other is the inter-carrier interference caused by mobility.

4.1 Peak to Average Power Ratio

In OFDM, the transmitted signal is obtained by taking the IFFT of the input data. The main issue with this technique is that some input combinations can lead to OFDM symbols with high amplitudes. To illustrate this, consider a long sequence of 1s to be transmitted. If the input to the IFFT is a sequence of ones, the output will be a delta function (i.e. the first value in the OFDM symbol is very high, followed by a sequence of zeros). The average power of the whole OFDM symbol is the same as other input combinations, but the peak power is a high value leading to high peak to average power ratio. In practical systems amplifiers are used in their linear region to amplify the signal. High peak to average power ratio means the amplifier must have a large linear region in order to avoid distortion when the amplifier reaches saturation. Amplifiers with such characteristics are usually very expensive, especially at high frequencies. To overcome this problem, coding can be used prior to the IFFT. The coding used aims at avoiding input combinations (e.g. long sequence of 1s) that will lead to OFDM symbols with high peak power [1].

4.2 Mobility

OFDM performs well in fixed and slowly varying channels. However, in fast fading channels, an irreducible error floor is encountered at high signal to noise ratios (SNR) due to inter-carrier-interference (ICI). In mobile links, if a transmitter sends a sinusoidal signal, its bandwidth becomes broader at the receiver due to the Doppler shift [3, 5, 6]. This shift in frequency is a minor issue in single carrier systems, but in

OFDM systems the orthogonality of the subcarriers (see Fig. 1) is destroyed by Doppler shift leading to interference between the carriers (ICI) [10, 12-16]. A mathematical model can easily be developed from Eq. (8). The channel is no longer constant within the OFDM symbol duration. However since OFDM is used in high data rate systems we can approximate the channel variation within an OFDM symbol by a linear equation as [14, 17]:

$$h(n,k) = h(0,k) + a(k)n \tag{13}$$

$h(0,k)$ is the channel response at the beginning of the OFDM symbol. $a(k)$ is the change (slope) in channel response at frequency k in one symbol period (T_s). We assume $a(k)$ is constant for the duration of an OFDM symbol. This approximation is valid for $f_D \times T_{OFDM} < 0.1$, where T_{OFDM} is the OFDM symbol duration and f_D is the Doppler shift.
Substituting (10) and (13) in (8) we get:

$$x(k) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{p=0}^{N-1} d(p).(h(0,p)+a(p)n).e^{j\frac{2\pi}{N}n.(p-k)} + W(k) \tag{14}$$

Expanding (14) for the desired subcarrier k we get:

$$\begin{aligned} x(k) &= \frac{1}{N}\sum_{n=0}^{N-1} d(k).(h(0,k)+a(k)n) \\ &+ \frac{1}{N}\sum_{n=0}^{N-1}\sum_{\substack{p=0 \\ p\neq k}}^{N-1} d(p).(h(0,p)+a(p)n).e^{j\frac{2\pi}{N}n.(p-k)} + W(k) \end{aligned} \tag{15}$$

Rearranging the terms in (15) we get:

$$\begin{aligned} x(k) &= d(k)\left[h(0,k) + \frac{N-1}{2}a(k)\right] + \frac{1}{N}\sum_{\substack{p=0 \\ p\neq k}}^{N-1} d(p).h(0,p).\delta(p-k) \\ &+ \frac{1}{N}\sum_{\substack{p=0 \\ p\neq k}}^{N-1} a(p)d(p).\sum_{n=0}^{N-1} n.e^{j\frac{2\pi}{N}n.(p-k)} + W(k) \end{aligned} \tag{16}$$

where the delta ($\delta(p\text{-}k)$) function is the Fourier Transform of a constant value. Noting that the delta function ($\delta(p\text{-}k)$) is zero except when $p = k$, (16) reduces to:

$$\begin{aligned} x(k) &= d(k)\left[h(0,k) + \frac{N-1}{2}a(k)\right] \\ &+ \frac{1}{N}\sum_{\substack{p=0 \\ p\neq k}}^{N-1} a(p)d(p).\sum_{n=0}^{N-1} n.e^{j\frac{2\pi}{N}n.(p-k)} + W(k) \end{aligned} \tag{17}$$

Let

$$h(k) = h(0,k) + \frac{N-1}{2}a(k) \tag{18}$$

and

$$C_{pk} = \frac{1}{N}\sum_{n=0}^{N-1} n.e^{j\frac{2\pi}{N}n(p-k)} \tag{19}$$

Substituting (18) and (19) in (17) we get:

$$x(k) = d(k).h(k) + \sum_{\substack{p=0 \\ p\neq k}}^{N-1} a(p)d(p).C_{pk} + W(k) \tag{20}$$

The first term on the right hand side of (20) is the desired signal while the second term is the the ICI. The value of C_{pk} affects the contribution of the subcarriers in ICI. Figs 4 and 5 show the imaginary and absolute square value of C_{pk} for k = 30 and 64-point FFT. The value of k shifts the curves to the left or right without changing the shape. The real part of C_{pk} was found to be ½ regardless of the subcarrier index except for $p = k$, i.e. the desired subcarrier, where its value is $(N\text{-}1)/2$ as can be seen from Eq. (17). The contribution of ICI mainly comes from the few adjacent subcarriers as observed from Figs. 4 and 5. By cancelling or equalizing the ICI contribution of these subcarriers, the interference can be reduced thus improving the performance of the OFDM system.

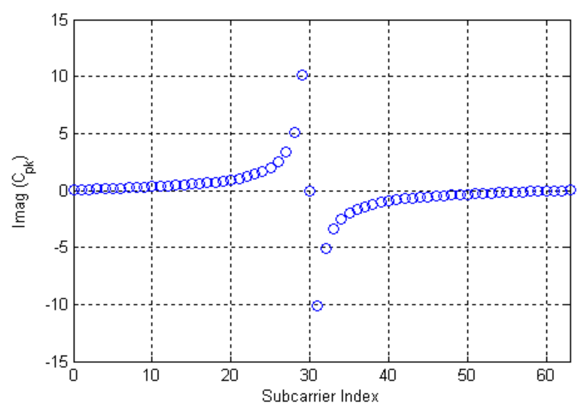

Fig. 4. Imaginary Part of Subcarrier Contribution (C_{pk}) to ICI

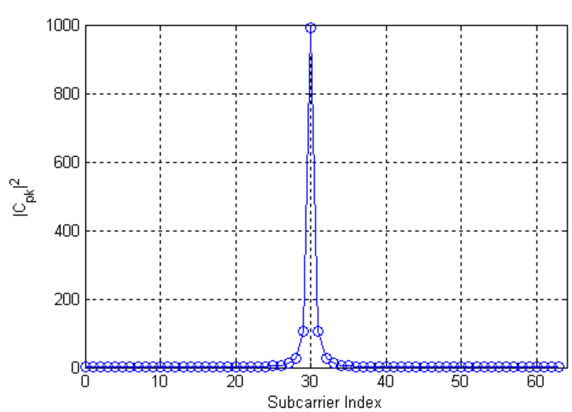

Fig. 5. $|C_{pk}|^2$ vs. Subcarrier Index for 64 Subcarriers and k = 30

The power of the received signal is given by:

$$E\left[|x(k)|^2\right]=E\left[\left|d(k).h(k)+\sum_{\substack{p=0\\p\neq k}}^{N-1}a(p)d(p).C_{pk}+W(k)\right|^2\right] \qquad (21)$$

where E(.) symbolizes the expected value. The data and the noise are usually uncorrelated. The noise is AWGN hence its autocorrelation function is [3, 7]:

$$E\left[|W(k)|^2\right]=E\left[W(k)\times W(k)^*\right]=P_N\delta(\tau) \qquad (22)$$

where P_N is the noise power and δ (.) is the Dirac delta function. If we assume the data in each subcarrier is independent and white, then the autocorrelation function of the data will be [17, 18]:

$$E\left[d(p)\times d(k)^*\right]=P\delta(p-k) \qquad (23)$$

P is the signal power. Eq. (21) hence becomes:

$$E\left[|x(k)|^2\right]=P\times E\left[|h(k)|^2\right] + E\left[\left(\sum_{\substack{p=0\\p\neq k}}^{N-1}a(p)d(p).C_{pk}\right)\times\left(\sum_{\substack{q=0\\q\neq k}}^{N-1}a^*(q)d^*(q).C^*_{qk}\right)\right]+E\left[|W(k)|^2\right] \qquad (24)$$

Using the white data assumption, eq. (24) reduces to:

$$E\left[|x(k)|^2\right]=P\times E\left[|h(k)|^2\right]+P\times\sum_{\substack{p=0\\p\neq k}}^{N-1}\left(E\left[|a(p)|^2\right]|C_{pk}|^2\right) + E\left[|W(k)|^2\right] \qquad (25)$$

The first term is the power of the desired signal, the second term is the power of the ICI, whereas the last term is the noise power.

The autocorrelation function of the channel in Rayleigh fading is given by [19]:

$$E\left[|h(k)|^2\right]=J_0(2\pi f_d\tau)=J_0(2\pi f_d mT_s) \qquad (26)$$

J_0 is the zero order Bessel function, f_d is the maximum Doppler shift, τ is the delay and mT_s is used instead of τ for digital systems to represent discrete time. The autocorrelation function of the slope of the channel ($a(p)$) is given by [14, 17, 19]:

$$E\left[|a(p)|^2\right]=-\frac{\partial^2 J_0(2\pi f_d mT_s)}{\partial m^2}\approx 2\pi f_d T_s \qquad (27)$$

Eq. (25) hence becomes:

$$E\left[|x(k)|^2\right]=P\times E\left[|h(k)|^2\right]+2\pi f_d T_s P\times\sum_{\substack{p=0\\p\neq k}}^{N-1}|C_{pk}|^2+P_N \qquad (28)$$

The ICI power is plotted in Fig. 6 for a fixed bandwidth ($1/T_s$) and various FFT sizes. As can be seen, the size of the FFT has a major impact on performance. At high speeds, use of a small number of subcarriers gives better performance since the OFDM symbol duration is short leading to smaller channel variation within the OFDM symbol. Moreover fewer subcarriers cause less interference. Both factors reduce the ICI power. The use of a small FFT size, however, leads to a wider bandwidth per subcarrier and, if the coherence bandwidth of the channel is small, the subcarriers may experience frequency selective fading and inter-symbol-interference (ISI) instead of flat fading. For low speeds and fixed links, use of more subcarriers is feasible to ensure a flat fading channel and possibly to provide Orthogonal Frequency Division Multiple Access (OFDMA) to serve a larger number of users.

4.3 Effects of ICI on BER

In this section we will investigate the BER performance of OFDM under mobility. We consider an OFDM system where a user is allocated a single subcarrier. To simplify the analysis we assume that all the users travel at the same speed. Although this is generally not the case, if we consider the maximum possible speed, the analysis will represent the worst case scenario. We also assume that the number of subcarriers is large such that the channel for each subcarrier can be considered to be flat fading. The probability of error of BPSK in a flat fading channel is given by [7]:

$$P_b=\frac{1}{2}\left(1-\sqrt{\frac{\gamma}{1+\gamma}}\right) \qquad (29.a)$$

$$\gamma=\frac{P\times E\left[|h(k)|^2\right]}{P_N} \qquad (29.b)$$

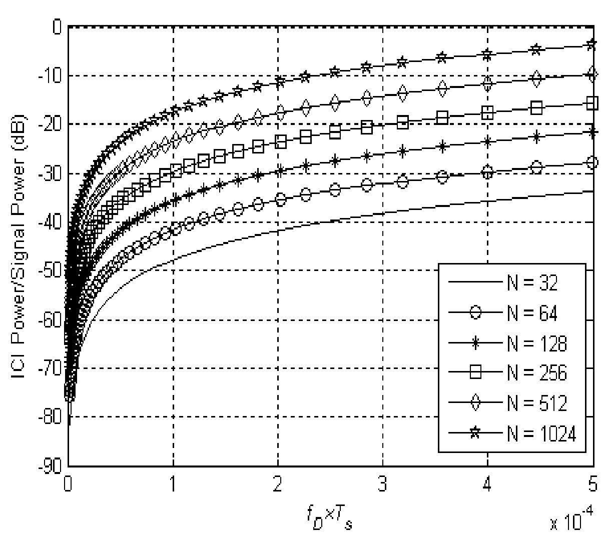

Fig. 6. ICI to Signal Power Ratio vs. Normalized Doppler Shift

For white data, the ICI will be similar to the white Gaussian noise, hence the overall noise power in a subcarrier in an OFDM system is given by:

$$SNR = \frac{P \times E\left[|h(k)|^2\right]}{2\pi f_d T_s P \times \sum_{\substack{p=0 \\ p\neq k}}^{N-1} |C_{pk}|^2 + P_N} \qquad (30)$$

Substituting (30) in (29.a), the probability of bit error is then found to be:

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{P \times E\left[|h(k)|^2\right]}{2\pi f_d T_s P \times \sum_{\substack{p=0 \\ p\neq k}}^{N-1} |C_{pk}|^2 + P_N + P \times E\left[|h(k)|^2\right]}}\right) \qquad (31)$$

Fig. 7 shows the probability of bit error for a 64-subcarriers, 10Mbps OFDM system with maximum Doppler shifts of 0, 5, 10, 50 and 100Hz. As we observe, increasing the Doppler shift (speed) degrades the performance of the system and, at high speeds, causes an irreducible error floor. Fig. 8 compares the performance of 10Mbps OFDM system with 64, 128, 256 and 512 subcarriers and 10Hz Doppler shift. It is clear that using a smaller number of subcarriers provides better performance since the ICI power is less compared to OFDM systems with large FFT.

4.4 ICI Mitigation Techniques

To reduce the effects of ICI several methods have been proposed, however, they can be generalized into two types. In the first type, it is up to the receiver to reduce ICI, the transmitter is not modified. The receiver uses an equalizer to reduce ICI. Several equalizers were proposed to work in the time (prior FFT) or the frequency (post FFT) domains [20, 21]. The equalizers improve the performance of the system but increase the complexity of the receiver. In the second type, the transmitter modifies the transmitted signal to reduce ICI. The transmitter for instance, may use only odd subcarriers for transmission and leave the even subcarriers unused thus reducing the number of interferers. A better method was proposed in [22] and [23], where the transmitter divides the subcarriers into groups. The same data is transmitted in all the subcarriers within a single group. However each subcarrier within a group is multiplied by a weighting factor. The factors are designed such that the sum of ICI from the subcarriers within the same group is zero. The receiver can demodulate the signal normally or can linearly combine the received signal from subcarriers with the same group to achieve better performance. The transmitter method has low complexity but sacrifices at least half of the bandwidth since at least half the subcarriers carry no data or duplicated data.

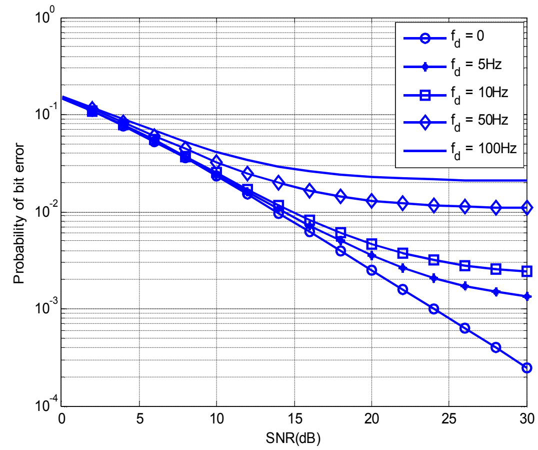

Fig. 7. Effect of Doppler Shift on Probability of Error (N = 64)

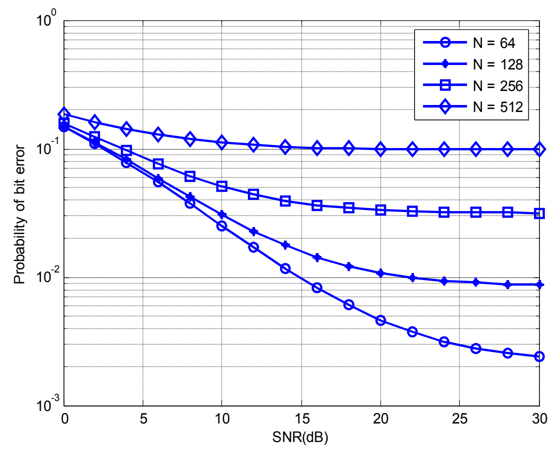

Fig. 8. Effect of No. of Subcarriers on Probability of Error (f_d = 10Hz)

5. CONCLUSIONS

In this paper we mathematically analyzed OFDM and investigated its performance in mobile links. OFDM provides high spectral efficiency by modulating the data over orthogonal subcarriers. A cyclic prefix is necessary at the end of each OFDM symbol to avoid inter-OFDM symbol interference. The OFDM signal has high peak to average power ratio, hence amplifiers with large linear amplification region must be used or coding should be employed to reduce this power ratio. In mobile links the channel variation destroys the orthogonality of the subcarriers and causes ICI. This leads to an irreducible error floor at high SNR. Equalization can be used to reduce ICI but it increases the receiver complexity. A low complexity method is to send the same data over several subcarriers to reduce ICI. This however, reduces the spectral efficiency.

REFERENCES

[1] J. R. Andrews, A. Ghosh, and R. Muhamed, *Fundamentals of WiMax*. New Jersey, USA: Prentice Hall Inc., 2007.

[2] IEEE, "IEEE Standard for Information technology— Telecommunications and information exchange between systems— Local and metropolitan area networks— Specific requirements," ed. New York, USA: IEEE, 2007.

[3] S. Haykin and M. Moher, *Modern Wireless Communications*. USA: Pearson Prentice Hall, 2005.

[4] B. Sklar. (July 1997) Rayleigh Fading Channels in Mobile Digital Communication Systems Part I: Characterization. *IEEE Communications Magazine*. 90-100.

[5] B. Sklar. (July 1997) Rayleigh Fading Channels in Mobile Digital Communication Systems Part II: Mitigation. *IEEE Communications Magazine*. 102-111.

[6] J. D. Parsons, *The Mobile Radio Propagation Channel*: John Wiley & Sons, 2001.

[7] J. G. Proakis, *Digital communications*, 4th ed. Singapore: McGraw Hill, 2001.

[8] M. C. D. Maddocks, "An Introduction to Digital Modulation and OFDM Techniques," BBC Research Department Report No RD 1993/101993.

[9] J. A. Stott, "The Effects of Frequency Errors in OFDM," BBC Reseearch Department Report No RD 1995/151995.

[10] Y. Chiu, D. Markovic, H. Tang, and N. Zhang, "OFDM Receiver Design," University of BerkeleyDec 2000.

[11] A. R. S. Bahai, *et al.*, *Multi-Carrier Digital Communications Theory and Applications of OFDM*, 2nd ed. USA: Springer, 2004.

[12] J. Cai, Z. Li, Y. Hao, and J. Cai, "Doppler Spread Estimation for Mobile OFDM Systems in Rayleigh Fading Channels," *IEEE Transactions on Consumer Electronics,* vol. 49, pp. 973-977, Nov 2003.

[13] B. Stantchev and G. Fettweis, "Time-Variant Distortions in OFDM," *IEEE Communications Letters,* vol. 4, pp. 312-314, Sep 2000.

[14] T. Wang, J. G. Proakis, E. Masry, and J. R. Zeidler, "Performance Degradation of OFDM Systems Due to Doppler Spreading," *IEEE Transactions On Wireless Communications,* vol. 5, pp. 1422-1432, June 2006.

[15] H. C. Wu, "Analysis and Characterization of Intercarrier and Interblock Interferences for Wireless Mobile OFDM Systems," *IEEE Transactions on Broadcasting,* vol. 52, pp. 203-210, June 2006.

[16] G. M. T. Abdalla, M. A. Abu-Rgheff, and S. M. Senouci, "Joint channel tracking and intercarrier-interference equalisation for vertical Bell Labs layered space time-orthogonal frequency division multiplexing in vehicle ad hoc network," *IET Intelligent Transport Systems,* vol. 3, pp. 409-418, March 2009.

[17] H. Meyr, M. Moeneclaey, and S. A. Fechtel, *Digital Communication Receivers*. USA: John Wiley & Sons, 1998.

[18] G. M. T. Abdalla, *et al.*, "Channel Tracking for VBLAST," *IEEE Vehicular Technology Magazine,* vol. 4, pp. 71-77, March 2009.

[19] W. C. Jakes, Ed., *Microwave Mobile Communications*. New York, USA: IEEE Press, 1994, p.^pp. Pages.

[20] W. G. Jeon, K. H. Chang, and Y. S. Cho, "An Equalization Technique for Orthogonal Frequency-Division Multiplexing Systems in Time-Variant Multipath Channels," *IEEE Transactions on Communications,* vol. 47, pp. 27-32, Jan 1999.

[21] W. S. Hou and B. S. Chen, "ICI Cancellation for OFDM Communication Systems in Time-Varying Multipath Fading Channels," *IEEE Transactions On Wireless Communications,* vol. 4, pp. 2100-2110, Sep. 2005.

[22] Y. Zhao and S.-G. Häggman, "Intercarrier Interference Self-Cancellation Scheme for OFDM Mobile Communication Systems," *IEEE Transactions on Communications,* vol. 49, pp. 1185-1191, July 2001.

[23] K. W. Park and Y. S. Cho, "An MIMO-OFDM Technique for High-Speed Mobile Channels," *IEEE Communications Letters,* vol. 9, pp. 604-606, July 2005.