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Comparison of Different Turbulence Models in Predicting Separated Flow over an Airfoil

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Abstract: Fluid flow around a NACA 4412 airfoil in a wind tunnel test section at Reynolds number of 3 x 106, based on the chord of the airfoil (150 mm) and free stream velocity (30 m/s), is considered. The study covers the boundary layers around the airfoil and the wake region at different angles of attack. Different turbulence models are used to predict separated flows over the airfoil. Two-equation turbulence models, k-ω and k-ε, and Reynolds Stress Model are tested for the ability to predict boundary layer separation on the airfoil. Reynolds Stress model captured the physics of separated flow favourably, and gave a very realistic evolution of the vortex formed due to separation. Statistics of the flow which is generated by RSM are in good agreement with an existing wind tunnel experimental data. The flow field which is generated by the two-equation turbulence models poorly predicted flow separation and vortex dynamics and consequently overestimated the lift coefficient for angles of attack larger than the critical angle of attack.

Keywords: Aerodynamics; CFD; RaNS; Two-equation model; RSM.

INTRODUCTION

Turbulent boundary layer separation from a surface is an important problem as it is responsible for setting an upper limit to the performance of aerodynamic devices. The maximum performance occurs near to the separation conditions.

1.1 Turbulence Models

Turbulence modelling is a key issue in most CFD simulations. All practical engineering flows are turbulent and hence need to be modelled.

RaNS-based turbulence models

The smart Reynolds decomposition has left us with the so called *closure problem* which means that the number of unknowns is greater than the number of equations; the additional unknowns are the Reynolds stresses which have to be modelled. For incompressible turbulent flow, all variables are divided into a mean part (time averaged) and fluctuating part. For the velocity vector this means that $\tilde{u}_i$ is divided into a mean part U_i and a fluctuating part u_i so that $\tilde{u}_i = U_i + u_i$. Time averaging yields:

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (1)$$

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial {x_j}^2} - \frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

where the turbulent stress tensor (also called *Reynolds stress tensor*) is given by:

$$\tau_{ij} = \overline{u_i u_j} \quad (3)$$

Two equation turbulence models

Two equations turbulence models include two additional transport equations (turbulent kinetic energy k-equation and turbulent dissipation ε-equation) to represent the turbulent properties of the flow. This allows the turbulence model to account for convection and diffusion of turbulent energy.

The k equation

The turbulent kinetic energy is the sum of all normal Reynolds stresses.

$$k = \frac{1}{2}\left(\overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2}\right) \quad (4)$$

k-equation is derived directly by setting the indices i = j in the equation that govern the Reynolds stresses, i.e.

$$\underbrace{\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j}}_{C} = \underbrace{-\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}}_{P} \underbrace{- \nu \frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}}_{\varepsilon} - \underbrace{\frac{\partial}{\partial x_j}\left\{u_j\left(\frac{p}{\rho} + \frac{1}{2}u_i u_i\right)\right\}}_{D} + \underbrace{\nu \frac{\partial^2 k}{\partial x_j \partial x_j}}_{D\ of\ k} \quad (5)$$

where C denotes convection, P denotes turbulent production, $\boldsymbol{\varepsilon}$denotes turbulent dissipation, and D denotes diffusion. The above equations can be symbolically written as follows:

$$C = P + \varepsilon + D \tag{6}$$

The ε equation

Two quantities are usually used in eddy-viscosity model to express the turbulent viscosity. In the $k - \varepsilon$ model, k and ε are used. The turbulent viscosity is then computed from

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \tag{7}$$

where $C_\mu = 0.09$. An exact equation for the transport equation for the dissipation $\varepsilon = \nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}$ can be derived, but it is very complicated and at the end many terms are found to be negligible. It is much easier to look at the $k$equation, and to setup a similar equation for ε. The transport equation should include a convective term, C , a diffusion term, D , a production term, P, and a dissipation term, ε, i.e.

$$C = P + D - \varepsilon \tag{8}$$

The production and dissipation terms in the k equation are used to formulate the corresponding terms in the ε equation. The terms in the k equation have the dimension $\partial k/\partial t \equiv [m^2/s^3]$, whereas the terms in the ε equation have the dimension $\partial \varepsilon/\partial t \equiv [m^2/s^4]$ .Hence, we must multiply P and ε by a quantity which has a dimension $[1/s]$. One quantity with this dimension is the mean velocity gradient which might be relevant to the production term, but not for the dissipation. A better choice should be $\varepsilon/k \equiv [1/s]$. Hence, we get

$$P - \varepsilon = \frac{\varepsilon}{k}(c_{\varepsilon 1} P - c_{\varepsilon 2}\varepsilon) \tag{9}$$

The final form of the ε transport equation reads

$$\frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\varepsilon}{k}(c_{\varepsilon 1} P - c_{\varepsilon 2}\varepsilon) + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial \varepsilon}{\partial x_j}\right) \tag{10}$$

Where, $\left(c_\mu, c_{\varepsilon 1}, c_{\varepsilon 2}, \sigma_k, \sigma_\varepsilon\right) = (0.09, 1.44, 1.92, 1, 1.3)$

Algebraic Reynolds Stress Model (ASM)

The Algebraic Reynolds Stress Model is a simplified Reynolds Stress Model. The RSM and $k - \varepsilon$ models are written in symbolic form as:

RSM : $C_{ij} - D_{ij} = P_{ij} + \Phi_{ij} - \varepsilon_{ij}$

$k - \varepsilon : C - D = P - \varepsilon$

In ASM we assume that the transport (convective and diffusive) of $\overline{u_i u_j}$ is related to that of k, i.e.

$$C_{ij} - D_{ij} = \frac{\overline{u_i u_j}}{k}(C - D)$$

Inserting the two previous equations into the equation above gives

$$P_{ij} + \Phi_{ij} - \varepsilon_{ij} = \frac{\overline{u_i u_j}}{k}(P - \varepsilon)$$

Thus the transport equation partial differential equations for $\overline{u_i u_j}$ have been transformed into an *algebraic* equation based on the assumption in the previous equations.

After re-writing these equations as equations for $\overline{u_i u_j}$ and inserting the models for Φ_{ij} and the isotropic model for ε_{ij} in the equation above and multiply by k/ε, we finally get

$$u_i u_j \left(\frac{2}{3}\delta_{ij}\right) k \ \frac{k}{\varepsilon}\frac{(1 - c_2)\left(P_{ij} - (\frac{2}{3}\delta_{ij})P\right) + \Phi_{ij,1w} + \Phi_{ij,2w}}{c_1 + \frac{P}{\varepsilon} - 1}$$

This model is an extension of the eddy-viscosity model where the c_μ constant is made a function of the ratio P/ε

Vorticity equation

Returning to viscous incompressible flow, The Navier-Stokes equations in vector form are given by:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u}.\nabla\vec{u} = -\nabla\left(\frac{p}{\rho} + gy\right) + \nu\nabla^2\vec{u} \tag{11}$$

By taking the curl of the Navier-Stokes equations we obtain the vorticity Eq. in details taking into account $\nabla \times \vec{u} \equiv \vec{\omega}$ we have

$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega}.\nabla)\vec{u} + \nu\nabla^2\vec{\omega} \tag{12}$$

$$\nabla \times (Navier - Stokes) \rightarrow$$

$$\nabla \times \frac{\partial \vec{u}}{\partial t} + \nabla \times (\vec{u}.\nabla\vec{u}) = -\nabla \times \nabla\left(\frac{p}{\rho} + gy\right) + \nabla \times (\nu\nabla^2\vec{u}) \tag{13}$$

The first term on the left side, for fixed reference frames, becomes

$$\nabla \times \frac{\partial \vec{u}}{\partial t} = \frac{\partial}{\partial t}(\nabla \times \vec{u}) = \frac{\partial \vec{\omega}}{\partial t} \tag{14}$$

In the same manner the last term on the right side becomes

$$\nabla \times (\nu\nabla^2\vec{u}) = \nu\nabla^2\vec{\omega} \tag{15}$$

Applying the identity $\nabla \times \nabla \cdot scalar = 0$ the pressure term vanishes, provided that the density is uniform

$$\nabla \times \left(\nabla\left(\frac{p}{\rho} + gy\right)\right) = 0 \tag{16}$$

The inertia term $\vec{u} \cdot \nabla\vec{u}$ can be rewritten as

$$\vec{u}.\nabla\vec{u} = \frac{1}{2}\nabla(\vec{u}.\vec{u}) - \vec{u}\times(\nabla\times\vec{u}) = \nabla\left(\frac{u^2}{2}\right) - \vec{u}\times\vec{\omega} \qquad (17)$$

where $u^2 \equiv |\vec{u}|^2 = \vec{u}.\vec{u}$

Then the second term on the left side can be rewritten as

$$\nabla\times(\vec{u}.\nabla)\vec{u} = (\vec{u}.\nabla)\vec{\omega} - (\vec{\omega}.\nabla)\vec{u} + \vec{\omega}(\nabla.\vec{u}) + \vec{u}(\nabla.\vec{\omega}) \qquad (18)$$

Putting everything together, we obtain the vorticity Eq.

$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega}.\nabla)\vec{u} + \nu\nabla^2\vec{\omega} \qquad (19)$$

COMPUTATIONAL SETUP

The inlet boundary velocity was set to 30 m/s for all turbulence models for direct comparison.

NACA 4412

Cord: 150 mm
Span: 300 mm
Re: 300,000

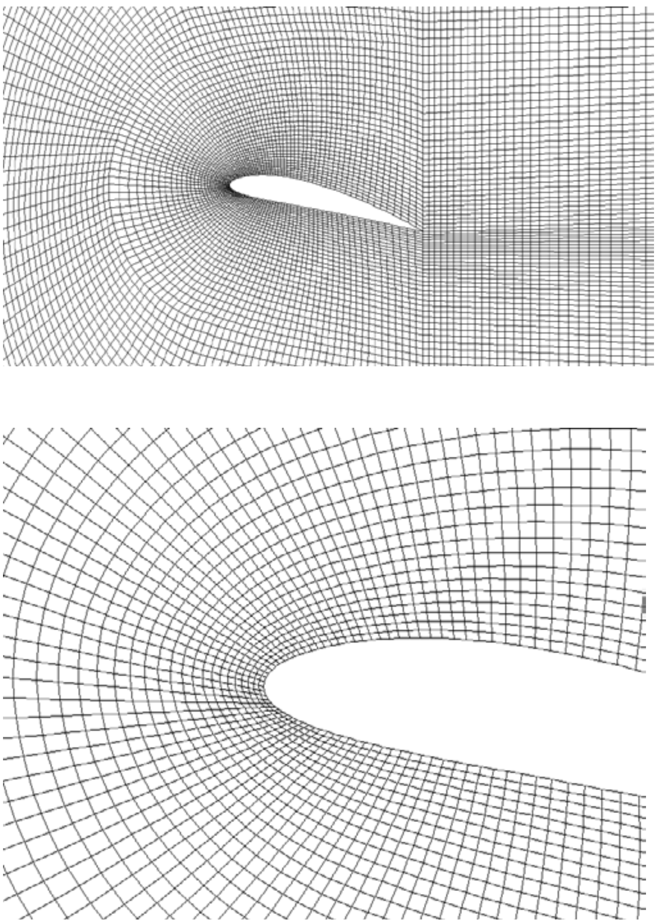


Fig. 1 Mesh around the airfoil

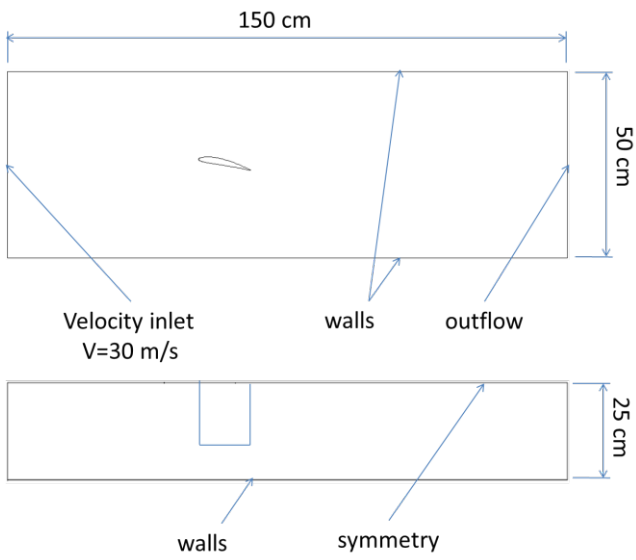


Fig. 2. Boundary Conditions

RESULTS AND DISCUSSION

Figs 3, 4, and 5 show the stream lines of the flow field which is generated by $k-\varepsilon$, $k-\omega$, and RSM models, respectively. The angle of attack is set to (α=24) so as to make sure that there exist boundary layer separation over the airfoil.

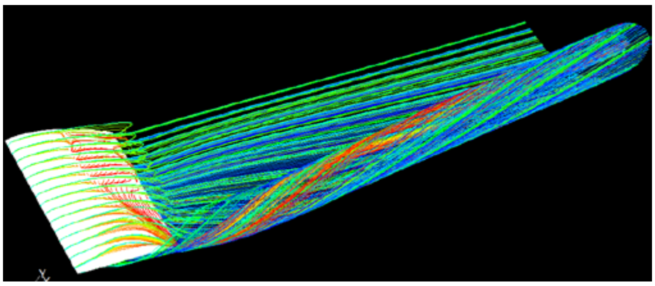


Fig. 3. Standard $\boldsymbol{k-\varepsilon}$

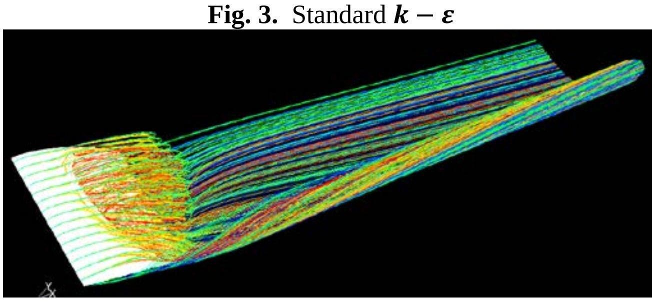


Fig. 4. Standard $k-\omega$

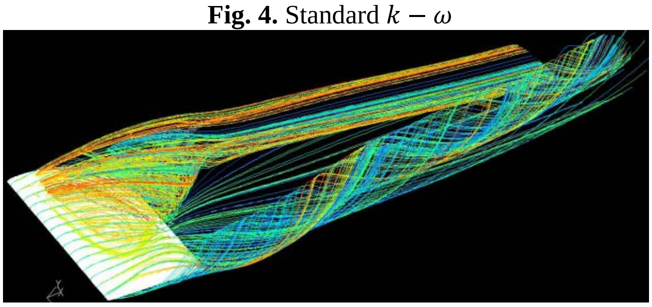


Fig. 5. RSM

Obviously, two-equation models could not predict the boundary layer separation on the top wall of the airfoil, whereas RSM nicely predicted the location and magnitude of the vortex formed due to the separation. Both models, two-equation and RSM, showed the tendency of the flow to form a wing tip vortex, but RSM model favourably predicted the shape of the vortex especially in the far wake region.

Properties at symmetry plane (RSM, α=24)

Figs 6, 7, and 8 show contours of static pressure, vorticity and a plot of streamlines together with static pressure contours. The relatively low static pressure over the airfoil creates unsteady separated flow and tip vortices.

Frame 1 to 12 show the evolution of the vortex on the upper wall of the airfoil. In the first seven frames the vortex is shaped, in Frames 8, 9, and 10 the vortex breakup, in frame 11 the vortex remaining flushed downstream to the far wake region by convection. In Frame 12 a new vortex is being formed in a life-cycle manner.

Pressure distribution

Figs 9, 10, and 11 show static pressure distribution over upper and lower surfaces of the airfoil versus distance from leading edge as calculated by $k-\varepsilon$, $k-\omega$, and *RSM* models.

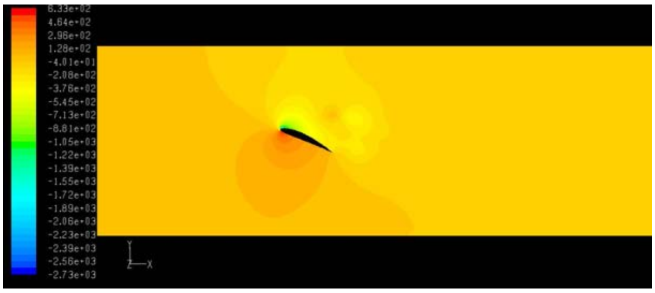


Fig. 6. Static Pressure

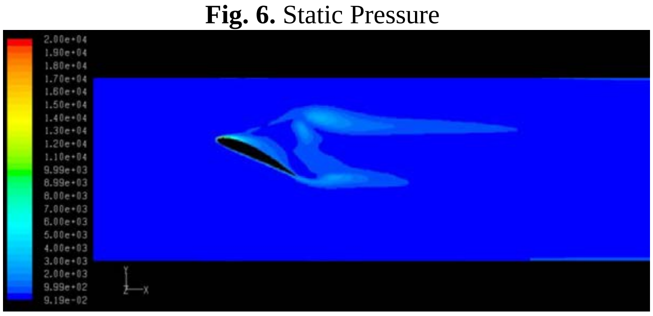


Fig. 7. Vorticity contours

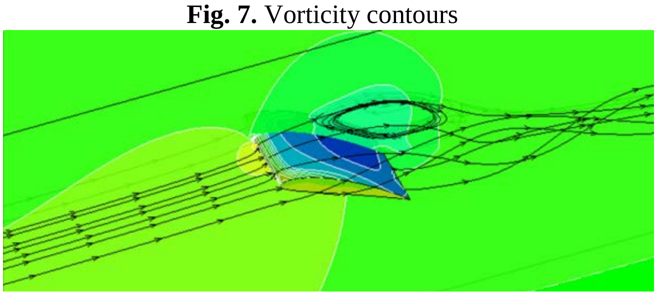
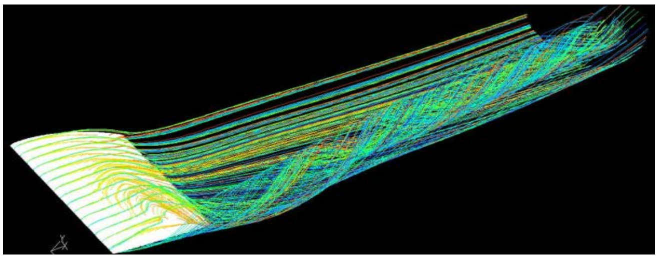
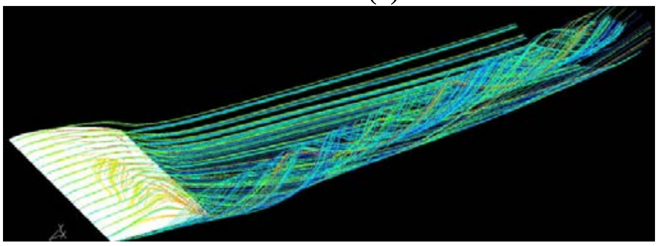


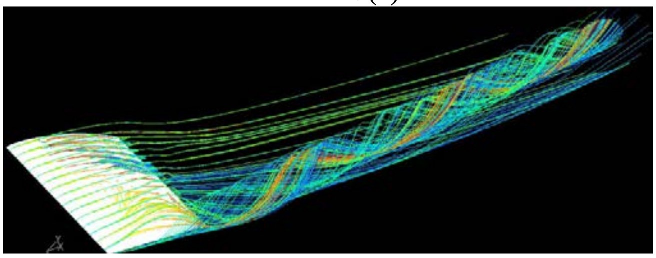
Fig. 8. Streamlines and static pressure



Frame No. (1)



Frame No. (2)



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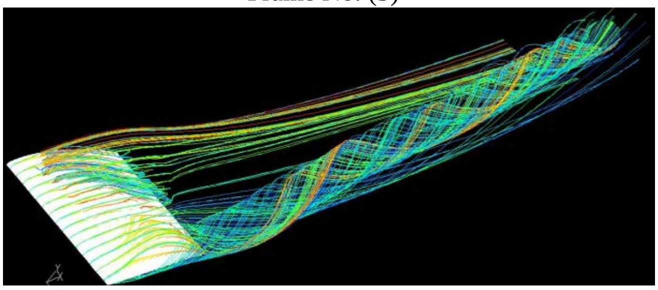

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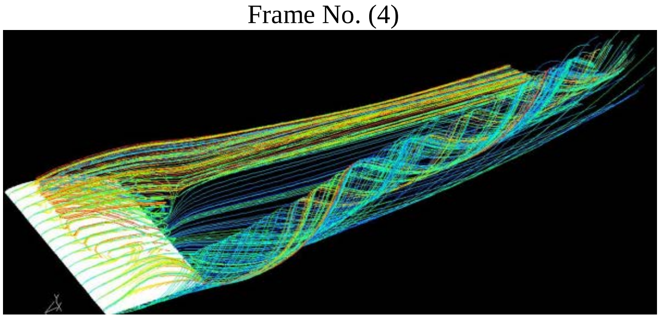
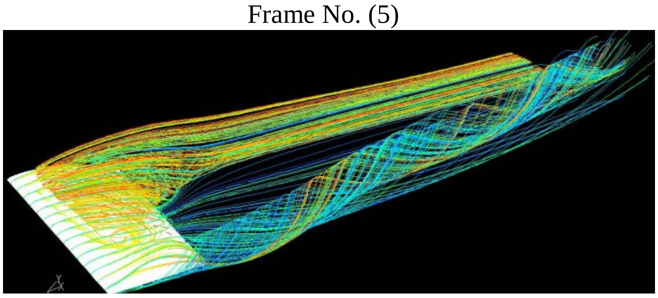


Frame No. (5)



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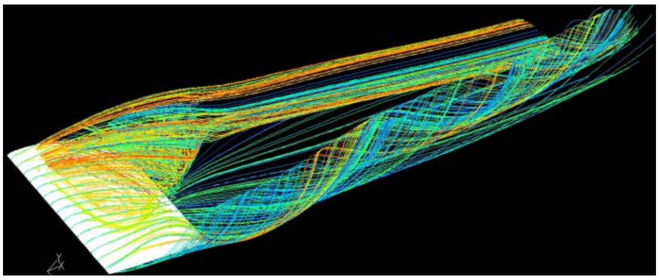

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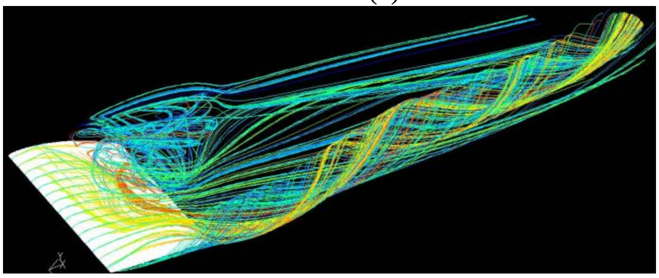

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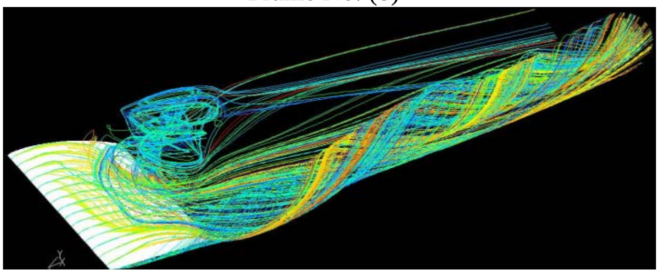

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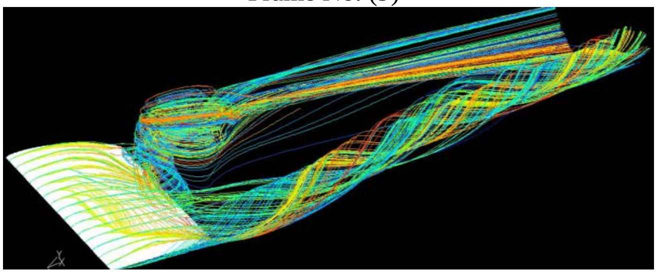
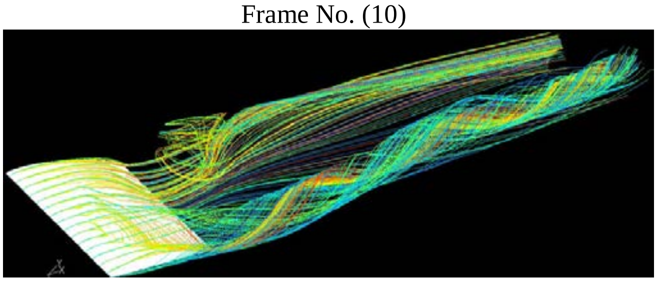
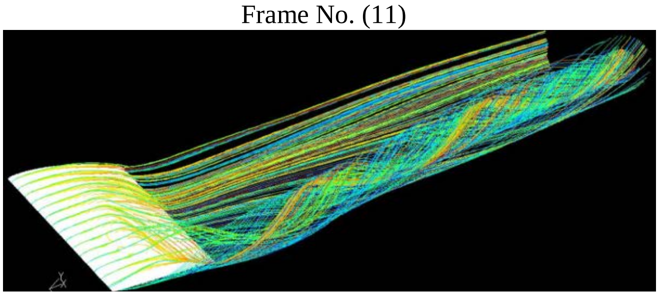


Frame No. (10)



Frame No. (11)



Frame No. (12)

Fig. 12 shows comparison of the lift coefficient calculated using the three models. Reynolds stress model predicted the boundary layer separation and consequently the declining of the lift force at large angles of attack. $\boldsymbol{k-\varepsilon}$ and $\boldsymbol{k-\omega}$

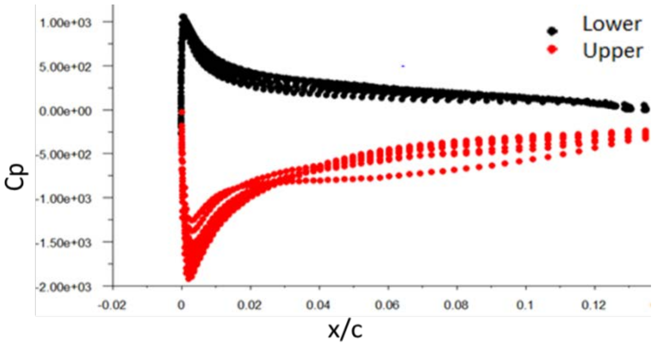


Fig. 9. Pressure coefficient vs. cord $(k-\varepsilon)$

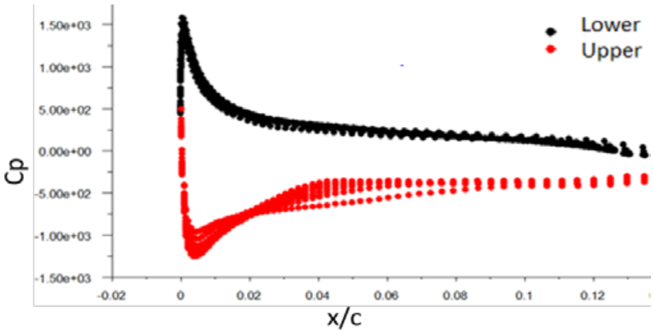


Fig. 10. Pressure coefficient vs. cord $(k-\omega)$

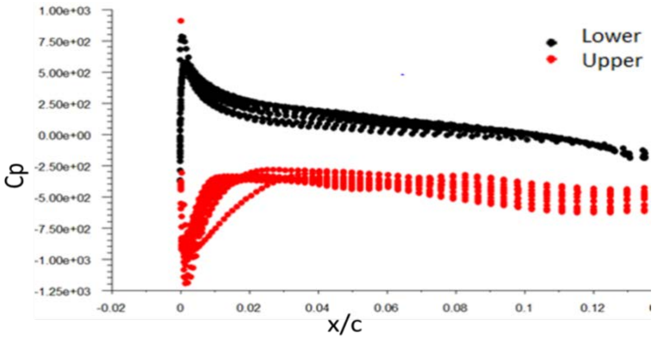


Fig. 11. Pressure coefficient vs. cord (RSM)

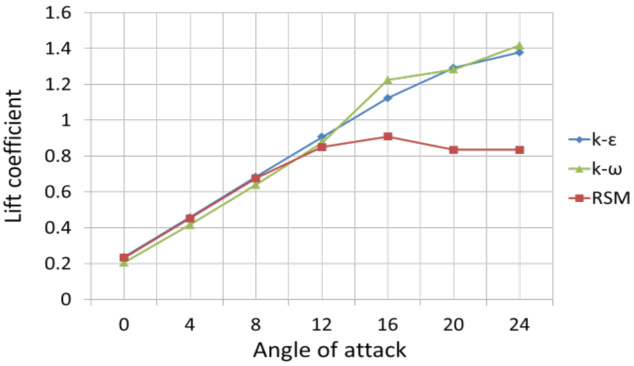


Fig. 12. Lift coefficient vs. angle of attack

models failed to predict the flow separation at large angle of attack, therefore, the lift coefficient calculated by these models continue to increase almost linearly without any trace of lift declination.

4. CONCLUSIONS

One of the most important aspects of a turbulence model for aerodynamic applications is its ability to accurately predict adverse pressure gradient boundary-layer flows. It is especially important that a model be able to predict the location of flow separation and the wake behaviour associated with it.

In this study, two-equation turbulence models, $\boldsymbol{k-\varepsilon}$ and $k-\omega$, and Reynolds Stress Modelwere tested for the ability to predict boundary layer separation on an air foil. Reynolds Stress model captured the physics of separated flow favourably giving very realistic evolution of the vortex formed due to separation. It was also found that lift force is highly correlated with flow separation, with the tremendous capabilities of RSM model in predicting the location and behaviour of a separated flow. In addition, RSM model generated lift coefficient that is comparable to the experimental data.

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