



Analytic Blackout Data for the Fully-Compensated Transmission Line

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Abstract: This paper presents a case for an analytical solution to the voltage stability of radial power links as determined by the maximum power transfer capability limit of a 100% capacitor-compensated transmission line. The prediction method uses an auxiliary circuit with variables that will simulate an active angle stability reference in the conventional sense of steady-state stability limits in radial links; but will not show in the final transfer characteristics of the source. The algebraic solution evolves in a voltage regulation curve that allows for dimensionless depiction of stability information including voltage limits for the stable transfer of maximum powers and hence for any powers below these maxima.

1. INTRODUCTION

Economic and environmental constraints for transmission expansion are dictating power systems operation closer to their nose point of the P-V curve, which, in conjunction with the increased advocating of transmission line compensation schemes for increased transfer capability, requires precise computation of steady-state stability limits. While the capacitive reactance in series capacitor installations is typically 25-75 percent of line inductive reactance, the existence of lines with 100 percent compensation has been reported [1, 2].

In this paper, results of a direct analytical solution to the voltage stability of radial power links as determined by their maximum power transfer capability limits are presented. The prediction method uses an auxiliary circuit at the terminals of a 100% capacitor-compensated transmission line with variables that will simulate an active angle stability reference in the conventional sense of steady-state stability limits in radial links; but will not show in the final transfer characteristics of the line. The algebraic solution evolves in a voltage regulation curve that allows for dimensionless depiction of stability information including voltage limits for the stable transfer of maximum powers and hence for any powers below these maxima.

2. The Angle Relation of Radial Power Links

The curves in Fig. 1 drawn for different ratios $\sigma = X_s/R_s$ of the circuit shown describe V-referenced maximum powers of equivalent capacitive reactance load impedance states; where conditions at any curve peak will coincide with those of conjugate impedance matching. The curves are based on

the following P- δ relation; where maximum received power is a state of critical transmission angle between E and V;

$$\text{viz. } \delta_c = \gamma = \tan^{-1} \frac{X_s}{R_s} ,$$

where

$$\begin{aligned} \alpha &= \tan^{-1} \frac{R_s}{X_s}, Z_s = \sqrt{R_s^2 + X_s^2} \\ P &= \frac{VE}{Z_s} \sin(\delta + \alpha) - \frac{V^2}{Z_s} \sin \alpha , \end{aligned} \quad (1)$$

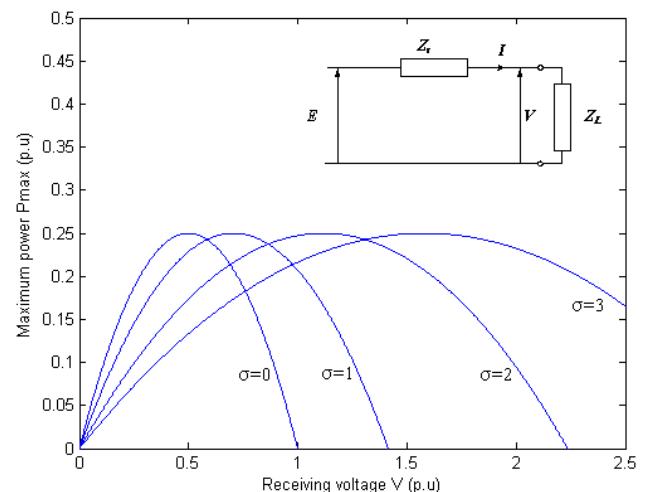


Fig. (1). Maximum radial link per-unit received powers for series reactance/resistance ratios

The upper-bound maximum powers use a per-unit system based on E and R_s that is adopted throughout; where E^2/R_s is base power, E/R_s is base current and V is in per unit.

In this paper, a case is considered for an analytical solution to the limits of maximum powers in radial lines as determined by terminal voltage conditions. This is the case of a line with 100 % series-capacitor-compensation for enhanced power transfer capability; which is analogous to the voltage-regulated resistive source, and for which the following per-unit terminal relations of impedance, power and reactive power under the condition can be written based on $\delta_c = 0$;

$$P = V - V^2, \quad Q = X_L = 0, \quad R_L = V/(1 - V) \quad (2)$$

The simple case is intended for comparison with the results to be presented for the voltage-controlled circuit based on the interesting observation that the maximum power curve occurs at unity power-factor for the range $0 \leq V \leq 1.0$, as the reactive power in this range is zero. This will be shown as representing a theoretical invalid state of the voltage-constrained stability condition.

3. Problem Statement and Solution Plan:

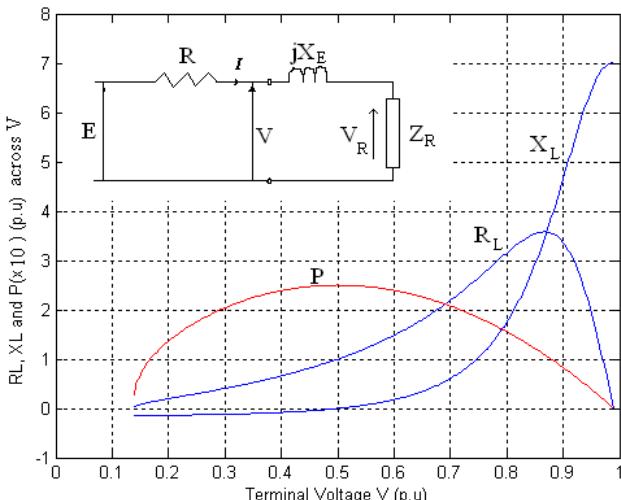


Fig. (2). Compensated link Angle-stability Simulator Circuit and associated power-maximizing load impedance constituents across the V terminals

The general plan for possibilities below the stated upper-bound maximums uses the variables of a terminated voltage-dependent reactive section X_E as shown in Fig. 2 (where the associated curves are to be shortly described). In this, V_R across Z_R will establish the necessary reference for critical transmission angle δ_c while the magnitude V_R is a variable. Stated in terms of impedance loading across the resistive source V terminals, where $R_L(V) = R_R$ and $X_L(V) = X_E - X_R$, based on *a priori* capacitive Z_R , the non-linear problem consists of identifying per-unit R_R , X_E , X_R and introduced δ that will satisfy the scalar identity for terminal voltage V of the following relation:

$$\bar{V} = \frac{R_L + jX_L}{1 + R_L + jX_L} \angle \delta \quad (3)$$

Under the condition, the angle of V with respect to induced V_R and; subsequently, the current angle with respect to V will come out as a result.

The simulator approach has originated in the course of analytic formulation to the voltage-constrained stability problem of radial power links with two internal node-voltage constraints. Whereas some of the solution aspects have been reported [3-8], further algebraic manipulations have resulted in a closed-form solution algorithm for the present single internal node circuit problem.

4. Solution Algorithm and Results:

This is based on a critical transmission angle δ_c that can be related to the reactive element of simulator circuit X_E and a no-load voltage state V_O (for induced $V_R = 0$) as follows:

$$X_E = \frac{V_0}{\sqrt{1 - V_0^2}} = \tan \delta_c \quad (4)$$

Fig. 2 gives a sample plot of load impedance constituents R_L , X_L and P_{MAX} generated using (the now linear) eqn. (3) for values of X_E and δ_c computed for the no-load voltage state $V_O = 0.99$ per-unit so as to produce identical power levels as in Fig. 1. Notice X_L transition at the 0.5 p.u. voltage point while traversing the lead-lag spectrum of load power factors over the range: $0 < V < 1.0$; which is proof of the reactive power demand in voltage-specified resistive sources. The 'inverted-V' power-factor data in Fig. 3 generated for a 100-percent series compensated line with initial no-load voltage $V_O=0.9$ p.u is normally read as the demand for reactive power Q brought about by changes in E that is required to achieve maximum transferable powers at any given terminal voltage V .

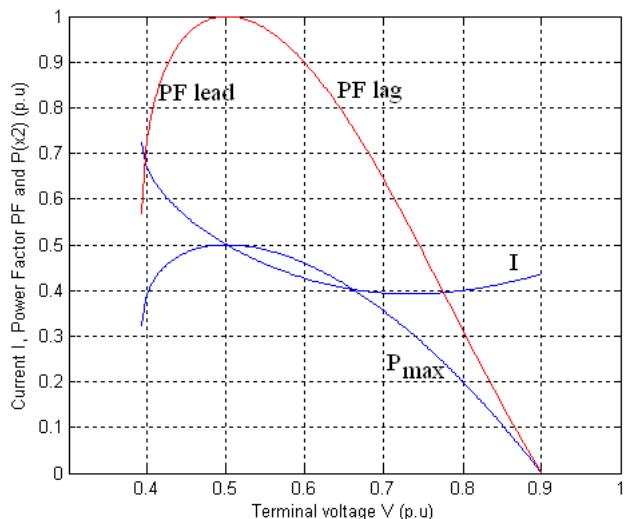


Fig. 3 Maximum-power Currents and Load Power-factor states for $V_O=0.9$ pu

In addition, algebraic manipulations aiming at simulator induced voltage V_R gave the following regulation characteristics in terms of V_O :

$$V_R = \sqrt{\frac{2V_0^4 - 2V_0^2 \sqrt{V_0^4 - V_0^2 + V^2} - V_0^2 + V^2}{1 - V_0^2}} \quad (5)$$

Fig. 4 shows a sample plot of V_R originating at different V_O that can be used in conjunction with δ_c to produce currents of subsequent power-maximizing load impedance functions for values of V below those specified for V_O ; as demonstrated by the steady-state stability information in Figs. 2, 3. Note function sensitivity to the upper limits of V_O and the evolving lower-bound voltage limits, indicating cut-off states for the stable transfer of power.

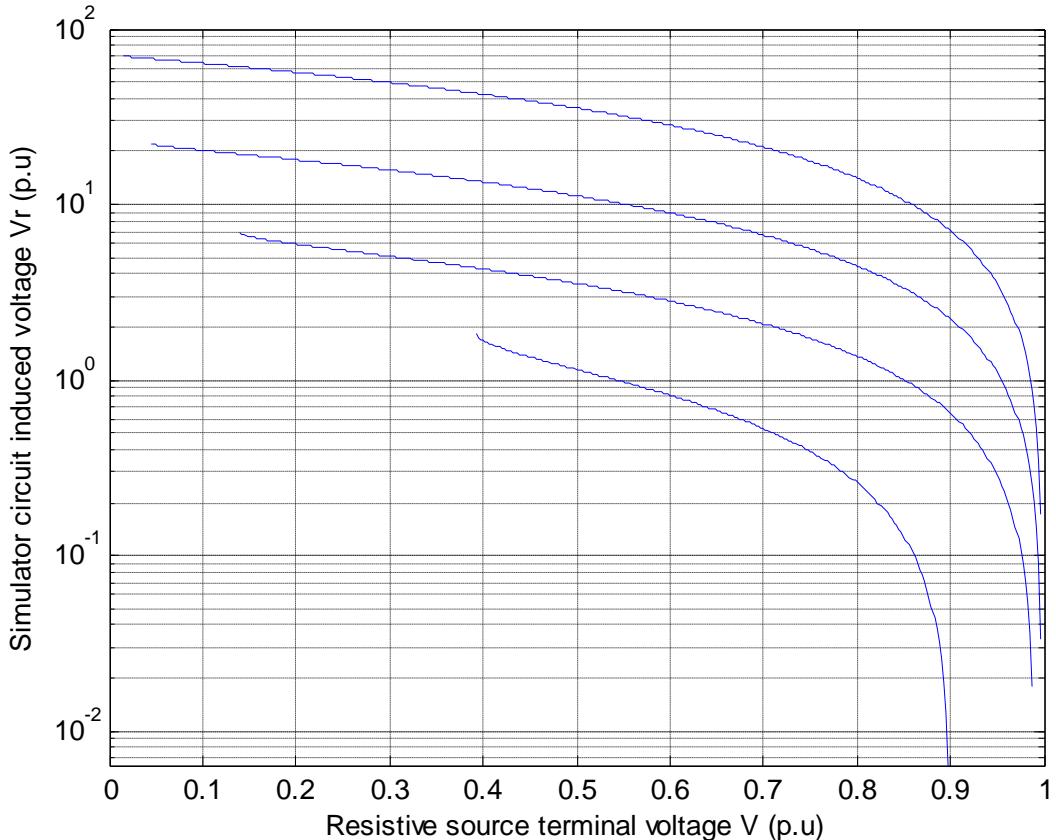


Fig. (4). Induced V_R Regulation curves originating at no-load voltages V_O (0.9, 0.99, 0.999, 0.9999 p.u) as a function of terminal voltage V

It must be stated that whereas the cases considered reveal continuous steady-state conditions for maximum powers, the maximizing function has been observed to embody certain singularity states. However, while the pattern of these states is being investigated, eqn. (5) provides information of function limits as may be noted from the following:

1. Exact identification of the limits to voltage stability is provided by curve plotting code; as the specification of V_O will automatically yield the lower-bound limit.
2. The equation radicands give a range for the specification of V_O within which the corresponding power curves will share a common occurrence at the peak power point of $V = 0.5$ per-unit, $P_{MAX} = 0.25$ per-unit. Equating the expression to zero gives the following for this range: $0.7071 \leq V_O < 1.0$. Note that

$V_O = 1.0$ p.u produces the stated invalid condition, and that the voltage range for maximum powers below the theoretical upper-bound down to zero decreases with reduced V_O as can be demonstrated using the regulation equation.

5. Conclusion:

The need for the precise computation of stability limits has intensified and the paper has demonstrated the ability to formulate algebraic solution of load conditions for maximum powers of a 100% capacitor-compensated transmission line as determined by terminal voltage states as well as the potential for dimensionless graphical depiction of the conditions.

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