Available online at www.uofk.edu
Proceedings Vol. 1 pp. 163- 166
Annual Conference of Postgraduate Studies and Scientific Research
17-20 February 2012-10-17 Friendship
Hall, Khartoum

A Load Impedance Function for the Simultaneous Maximum Voltage and Maximum Power

Fayez Mohammed EL-SADIK

Department of Electrical & Electronic Engineering, University of Khartoum, Khartoum, Sudan (Email: fmsadik@yahoo.com)

Abstract: Results of a load impedance function that will present conditions for the simultaneous occurrence of maximum voltage and maximum power at the terminals of a linear transducer circuit are given. The statement is based on algebraic solution to the problem of maximum power in voltage- and resistance-constrained radial transmission lines. As a result, it is shown that the load impedance conditions for maximum power with conjugate matching and Brainerd's maximum load voltage condition are embodied in the generalized load impedance function as the upper and lower limiting states of maximum received power; and that a purely resistive state for this function will determine the limit to stable power transfer. In addition, a case for the optimum transmission reactance that will enable extraction of maximum voltage at the maximum powers of conjugate impedance matching is demonstrated

Keywords: *Impedance matching, Load impedance function, Maximum voltage, Maximum power*.

1. INTRODUCTION

The theorems for maximum load voltage and maximum power transfer of systems represented by the circuit in Figure 1 were formulated on the basis of fixed magnitudes of either source voltage E or terminal voltage V according to the application for which the circuit is intended. In electronic engineering where E is normally fixed, the two statements are based on the variations of a passive load impedance circuit, with the occurrence of maximum power of E2/4RS for the conjugate load condition $\mathbf{Z}_L = \mathbf{Z}_s^*$ and maximum voltage [Brainerd 1] for the load condition ZL=-jZS2/XS. Notice that the conjugate-match shares with the V-constrained power system component the occurrence of angular separation between E and the resulting V that is concurrent with line transmission angle.

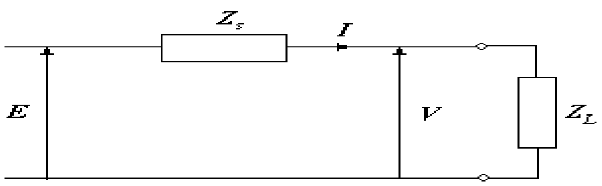


Figure (1). Transmission circuit configuration

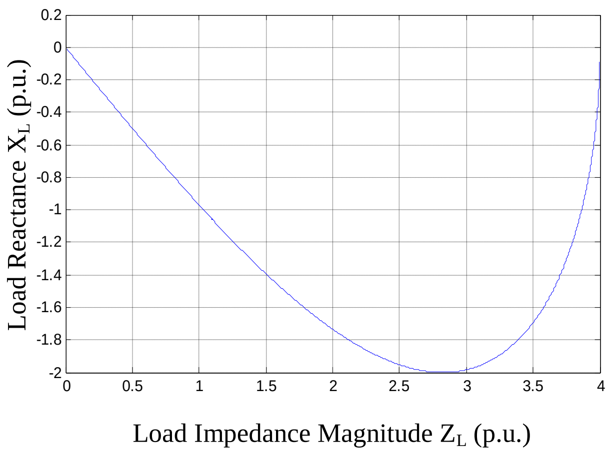


Figure (2). Base-case Load Impedance Function

In this paper, the results of an equation for a capacitive load impedance function with a pattern as depicted in Figure 2 are shown to evolve in conditions for the simultaneously occurrence of maximum voltage and maximum power whereby the conjugate condition represents the upper limit of maximum powers with the extreme lower limiting state of zero-power-maximums evolving in Brainerd's induction for the maximum load voltage condition. In addition, the voltage and power-constrained impedance function will introduce optimum conditions for transmission line reactance in a series-compensated application as well as conditions for voltage and power transfer cut-off as determined by the upper resistive state of this function.

2. Basis of the Voltage- and Power-Maximizing Load Impedance Function

The results to be presented of the generalized impedance function constitutes one of many equilibrium stability states of receiver voltage V that have evolved through algebraic formulation to the problem of steady-state stability limits in voltage-constrained radial transmission lines. While some of the solution aspects have been applied and reported elsewhere [2-7], attention is focused here on one of the states with a depiction of load impedance function as shown by the curve in Figure 2 that is plotted for a base-case transmission circuit configuration to be shortly analyzed. Note the implicit behavior of function per-unit active component R_L .

While details of the algebraic formulation would require prohibitively large space, a presentation is given here in graphical terms of easily-verifiable results for a case transmission circuit that will:

a. Provide proof of the conjugate match as a unique equilibrium state of transmission circuit reactance,

b. Demonstrate the simultaneous occurrence of maximum voltage and maximum power,

c. Demonstrate conditions for the stable voltage and power transfer cut-off as determined by a resistive load impedance state,

d. Provide proof of the maximum load voltage condition as a limiting state of load powers as these powers tend to zero.

3. Results for Maximum Voltage with Conjugate Matching and the effect of X_S

In describing the stability state of receiver voltage **V** in Figure 1, use is made of a per-unit system based on E and line resistance R_S such that the received power P is expressed as $P' = PR_S/E^2$ in *per-unit*. The corresponding actual values of line reactance X_S, load resistance R_L and load reactance X_L are accordingly modified to the R_S base so that $X_L' = X_L$ (actual)/R_S, $R_L' = R_L$ (actual)/R_S, and $X_L' = X_L$ (actual)/R_S in *per-unit*.

Based on this notation, it is possible to present in graphical terms the constituents of the load impedance function that will generate conditions for the concurrent maximum voltage and maximum power. This uses for verification a base case of unity E and unity line resistance R_S such that P is in *per unit* , and, in order to investigate line reactance effect, $Z_S = 1.0 + j\,X_S$ is the series impedance in per-unit.

Accordingly, the available transfer capability of the circuit gives $P_{max} = 0.25$ *per-unit* where the per-unit receiving end voltage will come out as a result in terms of X_S.

Now consider the voltage and power curves in Figure 3 for the case of a line with $Z_S = 1.0 + j\sqrt{3.0}$ *per-unit*. The behavior that has evolved as a result of applying the capacitive reactance and corresponding load resistance constituents of the load impedance depiction in Figure 2 shall reveal the following:

a. Maximum voltage of 1.0 p.u is synonymous with the maximum power of 0.25 p.u; both occurring at the load impedance magnitude $Z_L = 2.0$ p.u. and that,

b. The two curves will terminate at a specific terminal impedance $Z_L = 4.0$ p.u. as imposed by a zero state of capacitive load reactance.

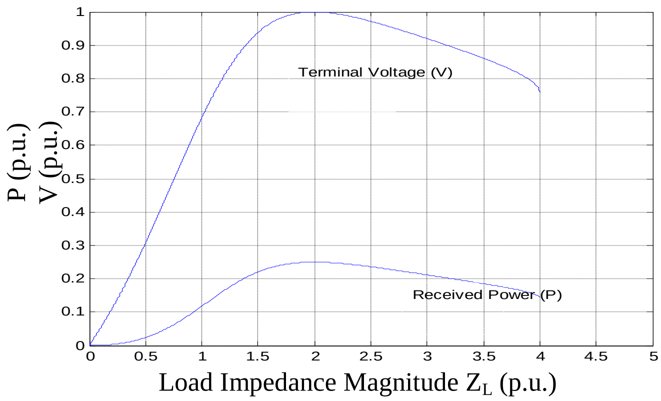


Figure (3). Base-case Voltages and Powers as a function of Load Impedance Magnitude:[$Z_S = 1.0 + j\sqrt{3.0}$ pu , E=1.0 pu]

The curves demonstrate the existence of analytic constituents of a capacitive load impedance function that will extract the conjugate-matched power which is concurrent with the maximum value of terminal voltage. In addition, the simultaneous discontinuities of voltages and powers shown is indication a purely resistive 'cut-off' impedance load above which no transfer of voltage and hence of power will take place.

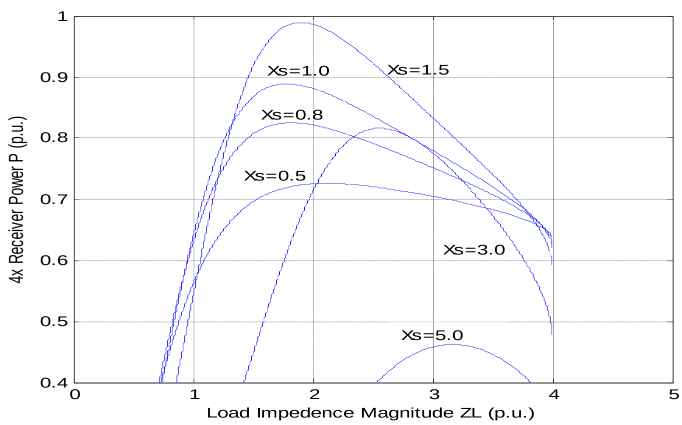
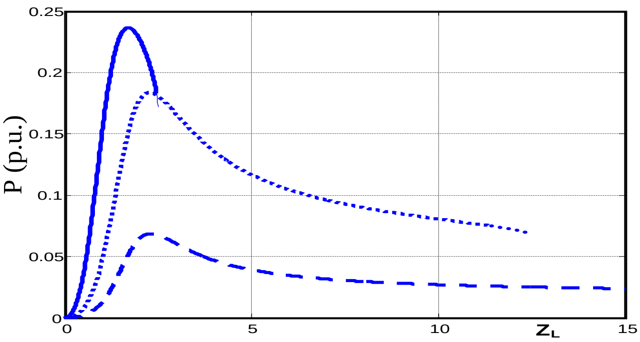


Figure (4). X_S effect on maximum power points

Figure 4 plotted for the influence of line reactance X_S on received powers provides further demonstration of the above-stated occurrences of maxima and cut-off limits if the corresponding voltage curves are generated using the maximizing load impedance constituents in Figure 2. Note that the pattern of these maxima will describe a locus where it can be seen as well as shown analytically that $X_S = \sqrt{3.0}$ *per-unit* as depicted in Figure 3 represents the optimum 'golden' reactance value (which is of the conjugate match) that will enable extraction of the maximum of maximums of received voltages and powers for the case considered. Such optimum reactance is sought in power transmission line compensation practice.

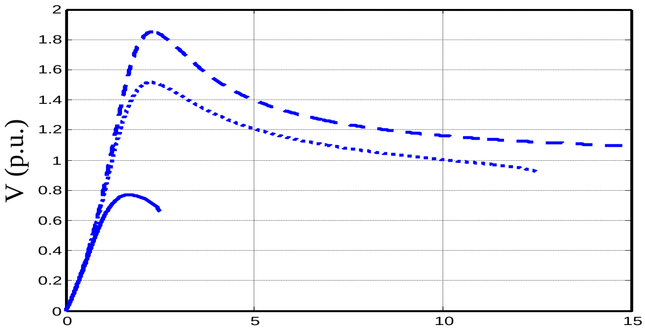
4. Constraints for Reduced Power and Voltage Maximums

In their generalized form, the two constituents of the maximizing load are both voltage and power-constrained such that these variables can be set *a'priori* as defining parameters for circuit terminal conditions. The sample set of power curves in Figure (5.a) and the corresponding load voltage curves in Figure (5.b) that demonstrate the influence of these parameters shows the natural tendency of terminal voltage increase with decreased load powers as well as the associated differences in the power transfer cut-off impedance limits. Notice the extended transmission cut-off range with decreased circuit powers. The curves are plotted for the base-case line reactance $X_S = \sqrt{3.0}$ *per-unit* and three different sets of impedance function constituents.



Load Impedance Magnitude Z_L (p.u.)

Figure (5).a



Load Impedance Magnitude Z_L (p.u.)

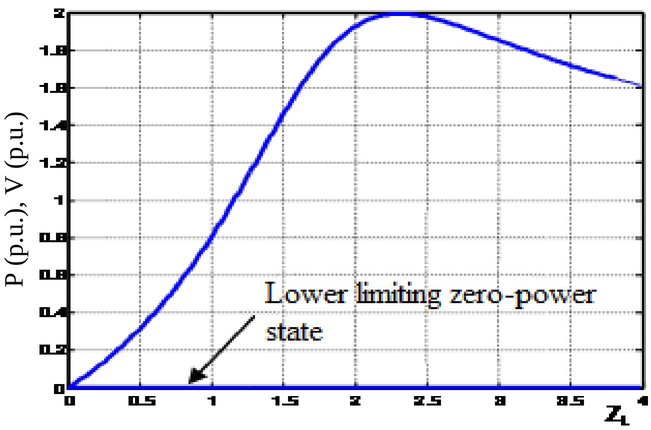
Figure (5).b

Figure 5. Results of three Load Impedance Function generating sets for base-case voltages at powers below that of conjugate matching

While because of space and time limitations algebraic formulation together with further deductions are left for a separate publication, a search for the resistance and reactance constituents of Z_L that will generate the power and associated voltage curves in Figure 5 for the case considered, while observing the occurrence of concurrent maximums and cut-off limits, is left for an algorithmic computer solution.

5. Brainerd's Voltage Condition as a Zero State of Maximum Powers

Noting the upper voltage and power points of conjugate matching in Figure 3 and the natural tendency of increased voltage with decreased power maximums exhibited in Figure 5, it is possible to deduce the terminal voltage conditions as the maximums of received powers tend to zero.



Load Impedance Magnitude Z_L (p.u.)

Figure (6). Base-case voltage behavior for the limiting zero-power states

As stated in his paper, while the conditions for current and power maximums are commonplace, specific problems of voltage maximization are solved by differentiation and that the condition for maximum voltage has evolved in a class discussion involving logical reasoning as to the physical nature of loads. In this regard Bernard Miller, a senior student in the Moore School, was credited for the discussion leading to the hypotheses which states that maximum voltage at the terminals of a passive linear transducer circuit is obtained when $Z_L = -jZ_s^2 / X_s$.

This can now be seen as evolving from the limiting zero-power conditions depicted in Figure 6 where a maximum voltage of 2.0 *per-unit* occurs at the load impedance value of $(4/\sqrt{3.0})$ *per-unit* for the case considered.

6. Conclusion

The load impedance characteristics that will simultaneously extract maximum powers at the maximum terminal voltages of a linear transmission circuit have been presented. As a result, it has been demonstrated in a case study that the well-known conjugate match represents a unique equilibrium state involving the amount of transmission circuit reactance. In addition, the non-linear characteristics of load constituents have introduced limiting conditions for the stable transfer of voltage and power as exemplified by a purely resistive cut-off state of the load impedance function.

7. Acknowledgement

The author whishes to acknowledge the assistance with manuscript preparation received from M. Elnourani and the departmental facilities provided.

REFERENCES

[1] Brainerd, J.G.: “Some General Resonance Relations and A Discussion of Thevenin’s Theorem”; Proceedings of the Institute of Radio Engineers, Volume 21, Number 7, July 1933

[2] El-Sadik, F.M., "Towards the Prediction of Power System Voltage Collapse", Sudan Engineering Society Journal, Vol. 47, No. 38, July 2000.

[3] EL-Sadik, F.M., “An Equation for the Coupling Resistance Influence on (The Many) Radial Link Power Reserves”, Proc. INREC 10, 1ST International Nuclear & Renewable Energy Conference, March 21-24, 2010, Amman, Jordan.

[4] EL-Sadik, F.M., “Results of Algebraic Equation for the Steady-State Stability Limits in Voltage-Constrained Transmission Systems”, Proc. CIGRE SC C4 2009 Kushiro Colloquium, June 7-13, 2009, Kushiro, Japan.

[5] El-Sadik, F.M., “Transmission Line Maximum Powers & Clarke Deduction for Reactance Networks”; Sudan Engineering Society Journal, Vol. 54, No. 51, Sep. 2008,

[6] EL-Sadik F.M., “Resistance Influence on the Synchronous and Compensating Reactances in a SMIB Power System”; IEEE Smartgrid, Proc. SPEEDAM 2010, Power Electronics Electrical Drives Automation and Motion, 2010 International Symposium on, Pisa, Italy.

[7] EL-Sadik, F.M.: "On the Specification of Bus Voltages and Radial Link Transfer Impedance Modes", 2010. CIGRE Canada Conference on Power Systems(program agenda), www.cigre.ca/agenda/agenda.pdf June 7-13, 2010, Vancouver, Canada.

[8] EL-Sadik, F.M.: 'A Relationship for the reactance behavior in voltage-constrained power systems components', IEEE Smartgrid , Proc. APPEEC 2011, Asia-Pacific Power and Energy Engineering Conference, 2011, Wuhan University, China.