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Study of Noise in Double and Triple Tunnel Junction Systems

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Abstract: Coulomb blockade (CB) and single electron tunnelling (SET) have attracted a lot of attention during the last two decades. Many applications have been proposed and many experimental and theoretical studies were reported. Successful exploitation of Single electron devices requires deep understanding of the physics governing the operation of these devices. Models based on quantum mechanical formulations have been tailored for the study of basic and simple structures with few tunnel junctions. Generalisations of such models are prohibitively cumbersome and have not been attempted. A general purpose semi-classical model has been derived and was successfully used in the study of noise properties in homogeneous long tunnel arrays. In this paper the semi-classical model is used to study the double junction system, known as the single electron transistor. The paper derives closed form finite frequency noise characteristics, together with the zero frequency Fano factor. The model is also used to analyse the dependence of the Fano factor against the applied voltages. It is shown that the IV characteristics and noise measures reveal rich details in the asymmetrical SET as compared to the symmetrical structure. The semi-classical model is also applied to the three-junction system. An analytical expression is derived for finite and zero frequency Fano factor. The model is also used to further the understanding of noise characteristics and the stability of such systems.

Keywords: *Shot noise; Fano factor; Coulomb blockade.*

1. INTRODUCTION

Correlated transfer of individual electrons resulting from the Coulomb blockade of electron tunneling in small structures have been observed in semiconductor nano-structures, metallic quantum dot devices and low-dimensional organic nanostructures, refer to [1] and [2] for a review. A single junction that is weakly coupled to a dc current source may generate single-electron tunneling oscillations with an average frequency *f=I/e*. A coupling resistance greater than the quantum resistance (h/e^2) is needed to control the continuous accumulation of charge on the junction capacitance before the tunnel event may occur.
An ensemble Monte-Carlo technique has been used to compute S(0) and hence the bandwidth of oscillations [3]. This technique was also used to compute the spectral density. Full counting statistics based on moments of generating functions have been used to study properties of shot noise in a simple single quantum dot system, i.e. a system of two tunnel junctions, e.g. [4]. In this paper, we use the semi-classical model presented in [5] to derive a an analytical expression Fano factor for the single electron transistor and the three junction system in the low applied voltage limit. The paper also investigates shot noise characteristics as a function of the applied voltage.

2. Single Electron Transistor

2.1 Two state model

At low applied voltages, electron transport through the SET is accomplished via two states. The electron tunnels into the middle node at a rate Γ_1 and leaves the node at a rate Γ_2. The distribution of time between successive tunnel events detected at one of the two junctions is expressed as a convolution of two Poisson processes and the Fourier transform of the time between events is therefore given as:

$$G_2(\omega) = \frac{\Gamma_1\Gamma_2}{(\Gamma_1 + j\omega)(\Gamma_2 + j\omega)} \qquad (1)$$

The frequency dependent Fano factor measured across a given tunnel junction in the system could be computed according to [1] as:

$$F(\omega) = \frac{1 - \left|G(\omega)^2\right|}{\left|1 - G(\omega)\right|^2} \qquad (2)$$

The Fano factor for in the low voltage, two state, mode of operation is therefore obtained as:

$$F_2(\omega) = \frac{\Gamma_1^2 + \Gamma_2^2 + \omega^2}{(\Gamma_1 + \Gamma_2)^2 + \omega^2} \qquad (3)$$

Equation [3] above is identical to the Fano factor obtained in [6]. Note that the zero-frequency Fano factor F(0) in above equation also agrees with [6]. It is noted the shot noise in the two state system is sub-Poissonian at all frequencies ($F(\omega) < 1$) and approaches the Poissonian limit at high frequencies. The minimum value at low frequencies is 0.5 for a system with symmetrical transition rates, i.e. $\Gamma_1 = \Gamma_2$. For a highly asymmetric system the shot noise approaches the Poissonian limit at all frequencies.

Increasing the voltage applied to the single electron transistor results in more states that contribute to the transport process. Using the formulation of [5] it is possible to compute the frequency dependent shot noise at any applied voltage.

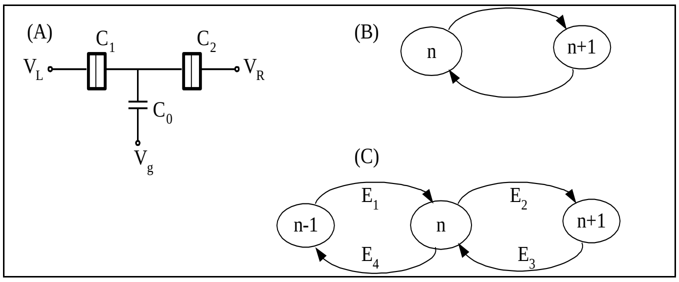

Figure (1). (A) Single electron transistor (B) Transport via two states & (C) transport via three states

2.2 Tri state model

When the voltage across the SET is increased, more states become available for tunneling during the conduction process. In this section we consider the situation when the system could be found in one of three states, these are labeled in Figure (1) as $\Psi = \{n\text{-}1, n, n+1\}$. Four events are needed to link the four states. The events are E_1, E_2, E_2 & E_4 and the corresponding tunnelling rates are λ_1, λ_2 λ_3 and λ_4 respectively. Assuming that the electrons enter from the left hand side and leave at the right hand side junction, we monitor the events across the right junction. The set of states where an electron will be able to tunnel is given by $\Psi_{init} = \{n+1, n\}$ and the resulting final states are $\Psi_{fin} = \{n, n\text{-}1\}$ linked by the events $E_{shot} = \{E_3, E_4\}$. To determine the distribution of the time between tunnel events across the junction we need to consider all sequences of events that could lead to two successive events in the list E_{shot}. Assuming that a tunnel event is detected across the junction at $t = t_0$, this event could either be E_3 or E_4.

Define the following path notation: Init Event→Final Event: {Events} to indicate the set of events needed to take the system from the initial state to the final state. Note that the last event leading to the final state will, by definition, result in a tunnel event across the junction under consideration.

Path H_1: $E_3 \rightarrow E_3$: $\{E_3, E_2, E_3\}$
Path H_2: $E_3 \rightarrow E_4$: $\{E_3, E_4\}$
Path H_3: $E_4 \rightarrow E_4$: $\{E_4, E_1, E_4\}$
Path H_4: $E_4 \rightarrow E_3$: $\{E_4, E_1, E_2, E_4\}$ (4) (3)

The Fourier transform of the time between successive tunnel events is computed [1] as:

$$G_3(\omega) = \sum_k q_k \prod_i \frac{\Gamma_i(k)}{\Gamma_i(k) + j\omega}$$

where q_k is the probability that successive tunnel events across a given junction follow a given path, H_k and $\Gamma_i(k)$ is the total tunnel rates of events at a given phase of the paths as defined by (4).

2.3 N- state model

If the voltage across the SET is increased further, the number of active states would increase. The additional states are known to be responsible for the staircase type increases of the I-V characteristics. The dynamics of transport could be represented by a birth-death model. It is possible to show that the number of possible paths between two randomly selected successive tunnel events across the left or the right junction is given as:

$$N_{paths} = \frac{N}{2}(N+1) - 2 \qquad (5)$$

where N is the number of active states in the system. The contribution of a given path to the overall density of the time between events could be computed in the same way as the tri-state model. Note that the contribution of a given path to the overall Fourier transform consists of two components, viz: $q_i G_i(\omega)$, where $q_i = p_k \prod_j \frac{\lambda_{j,m}}{\Gamma_j}$, p_k is the occupancy factor for the first state in the list, Γ_j is the sum of rates out of each state, and the product is for all items in the given path. The component $G_i(\omega)$ is expressed as:

$$G_i(\omega) = \prod_m \frac{\Gamma_m}{\Gamma_m + j\omega}$$

3. Double Dot Structure

At low applied voltages above the threshold voltage, the transport through a double quantum dot structure would result in three states, namely *{0,0}*, *{1,0}* & *{0,1}*. The tunneling rates between the three states are labeled Γ_1, Γ_2 & Γ_3 respectively. Transitions between states would follow a single path. The Fourier transform of time between events across any tunnel junction in this single-path system is obtained as [1]:

$$G_3(\omega) = \prod_{i=1}^{3} \frac{\Gamma_i}{\Gamma_i + j\omega} \qquad (6)$$

The frequency dependent Fano factor is derived using (2) above as:

$$F_3(\omega) = 1 - \frac{2\Gamma_1\Gamma_2\Gamma_3(\Gamma_1+\Gamma_2+\Gamma_3)}{(\Gamma_1\Gamma_2+\Gamma_1\Gamma_3+\Gamma_2\Gamma_3-\omega^2)^2+\omega^2(\Gamma_1+\Gamma_2+\Gamma_2)^2} \quad (7)$$

In [7], expression (7) is derived by solving the Master Equation in the first Coulomb staircase.

4. Results and Discussion

The simulator used in this study is described in [8]. This is a general purpose simulation tool based on the Master Equation formalism. An adaptive algorithm that searches for all paths contributing to the conduction process has been added to the simulator. The model was validated by computing the frequency dependent Fano factor at low voltages for the double and triple junction cases. It is shown than the simulator reproduces the Fano factors modeled in Eq. (3) and (7) respectively. In agreement with [6][7]. The formulations described above were used to study the noise properties of a metallic SET at low temperature. Co-tunneling is not considered in this study and a continuum of energy states is assumed at the quantum dots. Figure (2) shows the dependence of the zero frequency Fano factor on the applied voltage V_{ds}. The factor has a minimum value of 0.5 and approaches a constant value ~1 at the high V_{ds} voltage limit. SET asymmetry is introduced using a different tunnel resistances and capacitances. Figure (2) also shows the Fano factor for R_2= $10R_1$. The Fano factor decreases at the points where additional states are available for transport. With the additional states, electrons are able to get into the middle point faster than been able to leave the dot. The overall effect is an increased correlation of tunnel events. The Fano factor dependence on the applied voltage for the asymmetrical SET is consistent with the results of [7]. The number of states, events and total number of paths are consistent with Eq. (5).

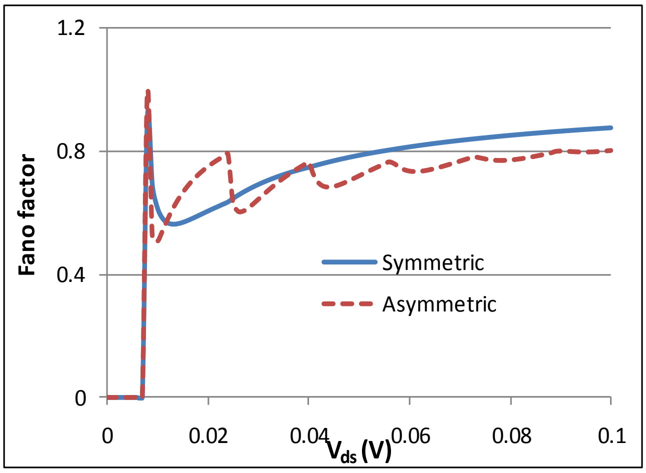

Figure (2). Fano factor as a function of applied terminal voltage for a SET with the following parameters: $C_1=C_2=10^{-18}$F, $R_1=R_2$=100K (Symmetrical structure) and $R_2=10R_1$ (Asymmetrical).

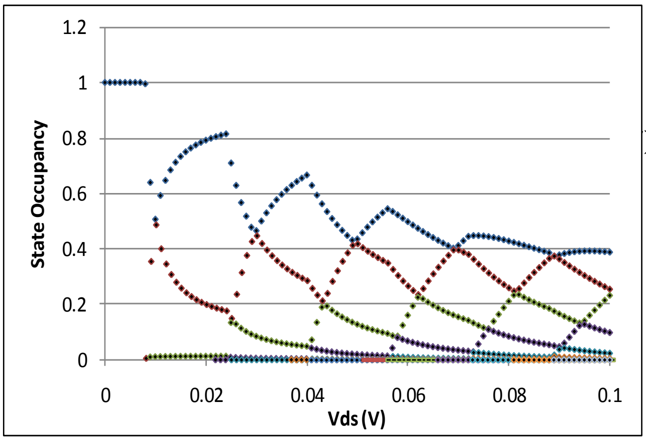

Figure (3). State occupation factors as a function of the applied voltage for the asymmetrical case. States correspond to n=0,1,..etc emerging from left to right.

The results presented in this study suggest that transport in this high voltage limit is not strictly dominated by only two states, but in fact is significantly affected by a large number of competing states. To check this argument we show in Figure (3) the occupation factors of the states and their dependence on the applied voltage. The figure shows a bundle of significant states that contribute to the transport process in the high applied voltage limit. Additional stochasticity is introduced by the random paths between two tunnel events detected at the measurement junction. The contribution of the different paths to the transport process could similarly be investigated. The overall effect of these two factors results in the shot noise approaching the Poissonian limit. We point out that shot noise is Possionian when the Coulomb blockade is lifted just above the threshold voltage. This agrees with the low voltage limit of Eq. (3). At very low voltages above the threshold level, only two states will contribute to the transport process. One of the two rates is known to be much lower than the second rate.

5. Conclusions

Using the formulation of [5] it is shown that analytical expressions for the Fano factor could be derived for two and three junction systems. For the two junction system, the changes in the Fano factor could be explained using the patterns of the electronic states and the resulting tunneling rates. It is shown that the emergence of new states and the decay of other states result in the Fano factor patterns observable in the asymmetrical SET case.

6. Acknowledgment

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