



Design, Development and Calibration of an Integrated Thin Ring Drawbar Pull Transducer for an Agricultural Tractor

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Abstract: The objective of this research work is to design, develop and calibrate an integrated thin ring drawbar transducer for measuring the horizontal drawbar pull of an agricultural tractor. The transducer is based on a thin proof ring that forms an integral part of the drawbar element with strain gauges bonded on the inside and the outside curvature of the proof ring. The strain gauges are arranged in a full bridge constant volt circuitry and interfaced to a data acquisition system. Static calibration tests on the transducer showed high degree of linearity between applied load and output volt with coefficient of correlation R^2 equal to 0.9968. The transducer measurements accuracy was within the acceptable range limits with measurement errors not more than 0.67% of the measured force magnitudes under the static measurements. The data acquisition system was able to successfully scan and recorded the transducer signals as programmed. The developed transducer can be a part of complete instrumentation system to be used for developing comprehensive information database on power and energy demand of various tractor and implement field operation in Sudan.

Keywords: Data Acquisition System; Transducer; Tractor; Implement; Strain gauges

1. INTRODUCTION

Sudan was one of the pioneering developing countries to introduce agricultural mechanization in the form of tractors and machinery to mechanize agricultural production in the irrigated and rain-fed sectors in the early forties of the last century. All over the years Sudan government has intervened to encourage increasing the cultivated area and thus increasing the agricultural production. There is a continuing need for adaptation of new technology and usage of appropriate machinery to achieve the set target. On one hand the costs of production using machines have tremendously increased to extend that some farmers have abandoned their farms. On the other hand, cultivation of large areas cannot be done without usage of agricultural tractors and machinery. Presently, with the advent of tractor assembly and partial manufacturing of some farm machinery especially at Giad Industrial complex and with the steady increase in the use of advance field irrigation and animal production systems, Sudan is expected to expand in the manufacturing of tractors and machinery components as well as the manufacturing of agricultural equipment in the near future. The technical specifications of these tractors and machinery cannot be made available to the farmers since these tractors and machinery were not properly tested for their performances in the true field conditions. Furthermore, local agricultural machinery distributors sell

imported machinery that had been developed for the crops and terrains in their country of origin. Trial and error modification works on these imported machineries were made by the distributors to comply with the local crop and terrain conditions. Testing and evaluation for all these agricultural machineries could not be made simply because of unavailability of testing and evaluation facility locally. Abroad, standard tractor tests such as the Nebraska Tractor Test and OECD (Organization for Economic Cooperation and Development) test are conducted on a special made track under specified standard test procedures and simulated test conditions similar to that of field conditions. The test results of the tractor are published and made available to interested parties upon request. The true field performances of the tractor-implement could vary with different tractor-implement operating conditions and field conditions. Such performances could only be known by having a system instrumentation that could monitor the needed performance parameters of the tractor-implement while the tractor-implement is running the operations in the field. To date there is no comprehensive agricultural information system or testing and evaluation facility for agricultural machinery in Sudan. Most test reports on imported farm machinery are from countries of origin, which are not able to indicate the actual tractor-implement performance in the field.

The practice of measuring tractor pull has been reported since the early part of last century by authors such as Sjogren [1] who is use mechanical transducer. The need for pull measurement arose with the advent of replacing horses with tractors as primary source of tractor power on North American Farms prompting the market to demand standardization of the tractor ratings. Out of this requirement came the development of (dynamometers) which initially merely consisted of calibrated springs. These were later improved by incorporating hydraulic element in order to enhance the portability of the measured mechanical signals [2]. Electrical force transducers were not adopted in the industry as quickly as their mechanical counter parts were. Interest in them only started to grow following a paper on stress analysis by Jensen [3]. Drawbar force is measured by transducer units inserting them in between the tractor and the implement, which introduces extra linkages that are not found in the hitch system during the normal field operation of the tractor [4]. Grevis-James et al. [5]. were able to overcome this difficulty by mounting strain gages in a hole drilled through the drawbar. However, such method of sensing suffered from low mechanical sensitivity and therefore requiring higher electrical amplification for better overall sensitivity. Zeorb and Musonda [6] developed an instrumented drawbar pin for draft measurement with trailed implement. This pin can only be used with in tractors with hitch pin holes larger than the select range. Musonda and Bigsby [7] modified integrated drawbar transducer to achieve mechanical amplification of strain and mounted strain gauges at appoint of peak strains. Kocher and summer [8] studied the design of drawbar transducers for measuring dynamic forces. McLaughlin et al. [9] developed a double extended octagonal ring drawbar transducer for 3D force measurements. The transducer utilized two extended octagonal ring transducers, one located in each side of the drawbar to measure the draft, vertical force, and side fore with minimum alteration of the tractor implement hitch point configuration. Kheiralla and Azmi Yahya [10] designed, developed and calibrated of a transducer for measuring the horizontal drawbar force of an agricultural tractor. Their the transducer design was based on a thick proof ring that forms an integral part of the drawbar element made from 7075-T6 aluminum alloy with strain gauges bonded on the inside and outside curvatures of the proof ring. The strain gauges are arranged in a full bridge constant current circuitry and interfaced to a acquisition system on board a tractor. Static calibration tests on their transducer showed high degree of linearity between applied load output strain with coefficient of correlation or R2 equal to 0.9997.

This paper describes design, development, and calibration a drawbar pull transducer as a part of complete instrumentation system to be used for developing comprehensive information database on power and energy demand of various tractor and implement field operation in Sudan.

2. MATERIAL AND METHODS

2.1 General Description

The design of drawbar pull transducer is based on a thin proof ring that forms an integral part of the drawbar element. This

machined transducer element replaces the existing drawbar of the tractor without affecting its original towing point. Fig. 1 illustrates the principal dimensions of the drawbar pull transducer. The drawbar element is of hitch pin type having one end fixed to the tractor body using two pins and other end free for hitching the implement using a single pin. The thin proof ring portion was made closed to the fixed end of the drawbar element to reduce lateral and longitudinal moments effect on the measurements. Mild steel being low in modulus of elasticity is employed in making the drawbar element to give greater strain sensitivity. The diametrical deflections of the ring are measured by the bonded strain gauges on the inside and outside curvatures of the proof ring. These strain gauges are arranged in a full bridge circuit and interfaced to the available data acquisition system on board a tractor. Fig. 2 illustrates the block diagram of complete system.

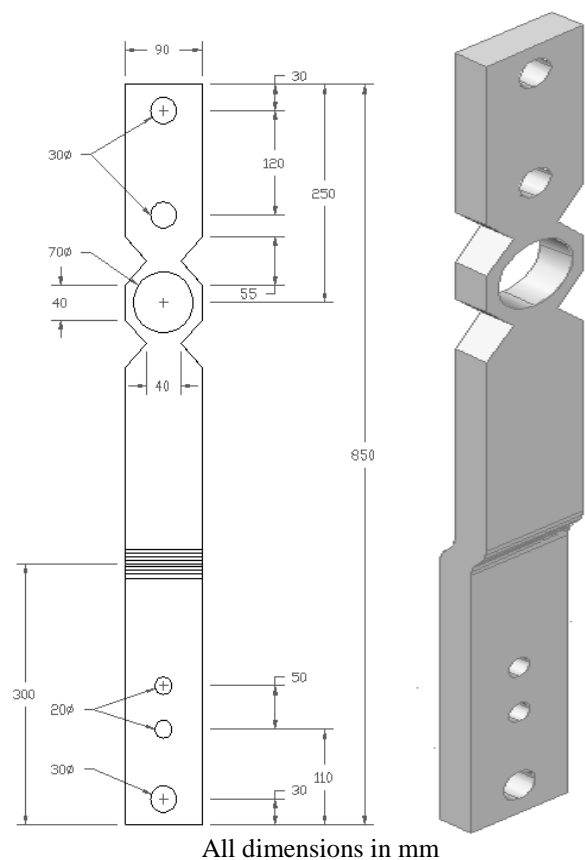


Fig (1). Principal dimensions of the drawbar pull transducer

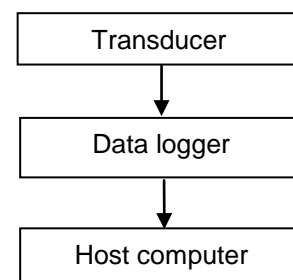


Figure (2). Block diagram of the data acquisition system

2.2 Thin Ring Analysis and Design

The thin proof ring portion of the drawbar element is designed to meet any external load that is below elastic limit. The basic principal of thing ring forces and elastic analysis was developed by Cook [11].

Consider one-half of a ring with the loads shown (see Fig. 3), where the top and bottom of the ring are restrained from rotation; M_0 is the moment required to satisfy this condition. The bending moment M_θ at any point in the ring is described as follows:

$$M_\theta = M_0 + \frac{Fr}{2} \sin \theta + \frac{Pr}{2} (1 - \cos \theta) \quad (1)$$

The total elastic energy in the ring is expressed by following equation:

$$U = \frac{1}{2EI} \int_0^\pi M_0^2 r d\theta \quad (2)$$

The angular rotation ϕ of ring at $\theta = 0$ is 0; thus

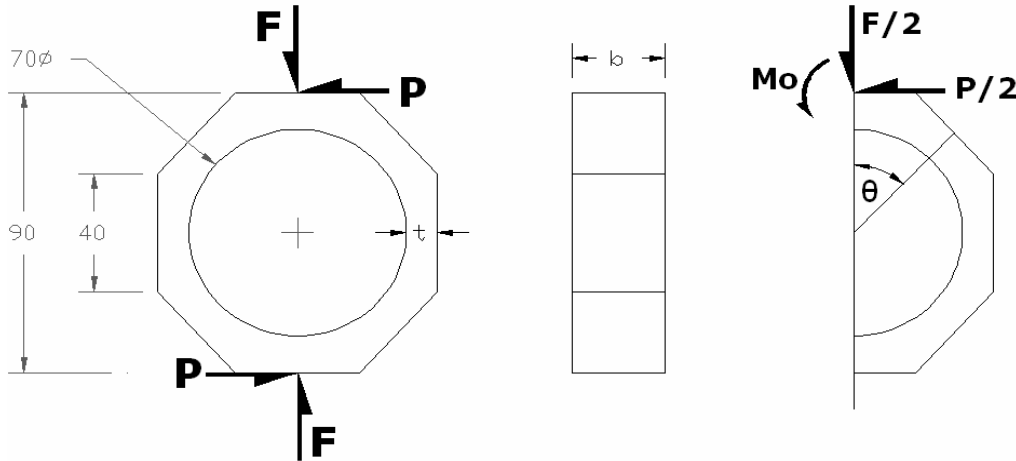


Figure (3). A thin ring's forces and elastic analysis

The two positions $\theta = 39.6^\circ$ and $\theta = 90^\circ$ are each a strain node for one force. Hence the moment due to vertical and horizontal force can be computed as:

$$M_{39.6^\circ} = -\frac{Pr}{2} \cos 39.6^\circ = -0.385 Pr, \quad (10)$$

$$M_{90^\circ} = \frac{Fr}{2} \left(\sin 90^\circ - \frac{2}{\pi} \right) = +0.181 Fr. \quad (11)$$

The strain ϵ in a thin ring is computed by the following equation:

$$\epsilon = \frac{6M}{Ebt^2}. \quad (12)$$

Therefore, strains due to vertical and horizontal force can be computed as:

$$\left(\frac{\partial U}{\partial M_0} \right)_{\theta=0} = 0 = \frac{1}{EI} \int_0^\pi M_\theta \frac{\partial M_\theta}{\partial M_0} r d\theta, \quad (3)$$

$$0 = \int_0^\pi \left[M_0 + \frac{Fr \sin \theta}{2} + \frac{Pr}{2} (1 - \cos \theta) \right] d\theta. \quad (4)$$

when integrated this gives

$$M_0 \pi + Fr + \frac{Pr \pi}{2} = 0 \quad (5)$$

Eliminating and rearranging this gives

$$M_0 = -\frac{Fr}{\pi} - \frac{Pr}{2}. \quad (6)$$

Substituting M_0 in equation (1), it yields the following equation:

$$M_\theta = \frac{Fr}{2} \left(\sin \theta - \frac{2}{\pi} \right) - \frac{Pr}{2} \cos \theta. \quad (7)$$

Knowing that the moment due to $F/2$ is zero when

$$\sin \theta = \frac{2}{\pi}, \theta = 39.6^\circ, \quad (8)$$

Also, the moment due to $P/2$ is zero when

$$\cos \theta = 0, \theta = 90^\circ \quad (9)$$

$$\epsilon_{39.6^\circ} = 2.31 \frac{Pr}{Ebt^2} \quad (13)$$

$$\epsilon_{90^\circ} = 1.09 \frac{Fr}{Ebt^2}. \quad (14)$$

where F is the applied horizontal force in kN, P is applied vertical force in kN, b is the width in mm, r is the mean radius in mm, t is the thickness in mm.

The available maximum drawbar power for a tractor in accordance to ASAE D497 is estimated to be 75 to 81% of its net engine power (ASAE 1996). The maximum drawbar pull of the tractor is calculated as:

$$F = DP \times 3.6 / S \quad (15)$$

where F is applied load(Pull) in kN, DP is the drawbar power. The drawbar pull transducer is to be developed for the Massey Ferguson 290 tractor with rated engine power of 61 kW. The tractor when operating at the lowest gear combinations (i.e. operating at 2.5 km/hr) can develop a maximum pull of 50 kN. As on this basis, a design load of 50 kN is used in the design analysis, but only 20 kN was considered as common operating load in field for design of the thin proof ring portion of the drawbar element. Various combinations of internal radius, external radius and height dimensions are being tried for the computation of tensile stresses in the thin ring design. Dimensions of 35, 30 and 10 mm for mean radius, width and thickness, respectively, are finally chosen for the design of this thin proof ring on the basis of the material strength and practical constraint. Based on the selected design dimensions, the maximum tensile stress calculated to be 254.33 MPa. This stress is below the elastic stress for mild steel.

The drawbar transducer element had strain gauges bonded on the inside and outside of the ring curvature at 90 degrees locations. The strain gauges are of Kyowa KFG-10-120-C1-11L1M2R type with gauge resistance of 120 ± 0.4 and gauge factor of 2.1 ± 1.0 %. The strain gauges are ready provided with 1-meter length lead wires and not required additional soldering. Easy installation and good signal transmission are the main reasons of employing such gauge type. The available four gauges are wired to a 5 Volts Wheatstone bridge configuration with output volt described by Daily and Riley [12] as:

$$\Delta E = \frac{IR_1R_3}{\sum R + \sum \Delta R} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} + \frac{\Delta R_1\Delta R_3}{R_1R_3} - \frac{\Delta R_2\Delta R_4}{R_2R_4} \right) \quad (16)$$

Where $\sum R = R_1 + R_2 + R_3 + R_4$

and $\sum \Delta R = \Delta R_1 + \Delta R_2 + \Delta R_3 + \Delta R_4$

equation (16) shows that the output signal ΔE is nonlinear with respect to ΔR because of the term $\sum \Delta R$ in the denominator and because of the second- order terms in the numerator. In practice, when measuring elastic strains in metals using properly designed Wheatstone bridge, the nonlinear terms can be made in significant. Therefore the much simpler and very good approximate relationship can be written in terms of strain, since change in the resistance $\frac{\Delta R}{R} = G_F \epsilon$ and with like gauges in all four bridge arms,

the equation (16) can be written as

$$\Delta E = \frac{IR_g G_F}{4} (\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) \quad (17)$$

where ΔR is the change in gauge resistance in ohm, R is the initial gauge resistance in ohm and ϵ is strain in micro-strain and G_F is the gauge factor, is an index of the strain sensitivity of the gauge.

Then sensitivity of the transducer is expressed in terms bridge output mvolt per unit force applied. The predicated channel sensitivity for the data acquisition system is calculated to be 0.113 m volt/kN.

in kW and S is the travel speed in km/hr.

2.2 Drawbar Transducer Deflection

Consideration is made in the design analysis to assure small deflection of the thin proof ring under the design load.

The horizontal (δ_p) deflection of the ring can be computed as follows:

$$\delta_p = \frac{\partial U}{\partial (P/2)} = \frac{1}{EI} \int_0^\pi M_\theta \frac{\partial M_\theta}{\partial (P/2)} r d\theta, \quad (18)$$

$$\delta_p = \frac{1}{EI} \int_0^\pi \left[\frac{Fr}{2} \left(\sin \theta - \frac{2}{\pi} \right) - \frac{Pr}{2} \cos \theta \right] (-r \cos \theta) r d\theta. \quad (19)$$

$$\text{hence} \quad \delta_p = 9.42 \frac{Pr^3}{Ebt^3}. \quad (20)$$

Similarly, the vertical deflection can be computed as follows:

$$\delta_f = 1.79 \frac{Fr^3}{Ebt^3}. \quad (21)$$

Knowing various dimensions and material property, the drawbar pull transducer deflection at the designed load is calculated to be 0.38 mm .

2.3 Dynamic Measurement of Transducer

The transducer under dynamic measurement response can be represented as a cantilever beam under a simple harmonic vibration that is excited by a fluctuating load at its free end. This can be modeled as a spring-mass-damper system with a single degree of freedom as indicated in Fig. 4. The general equation for the system is expressed by a second order differential equation subjected to an arbitrary applied force $F(t)$ as follows:

$$m \frac{d^2 x}{dt^2} + C \frac{dx}{dt} + kx = F(t) \quad (22)$$

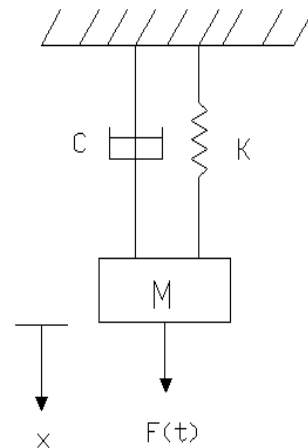


Figure (4). Drawbar pull transducer modeled as mass-spring-damper system

For a system oscillating without applied force and damping, the equation can be simplified as:

$$m \frac{d^2 x}{dt^2} + kx = 0 \quad (23)$$

The general solution for any homogeneous differential equation similarly to equation (23) is expressed as:

$$x = C_1 \sin(\omega t + \theta) \quad (24)$$

The free un damped angular frequency of the system is given as:

$$\omega = \sqrt{\frac{k}{m}} \quad (25)$$

Since the natural frequency and angular frequency can be related by $f = \omega \sqrt{2\pi}$ and stiffness related to deflection by $k = mg / \delta$, the transducer natural frequency can be expressed in terms of gravity acceleration and deflection and is given as:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta_f}} \quad (26)$$

where f is the natural frequency in Hz , δ_f is the static deflection in mm and g is acceleration due to gravity in m/s^2 . Knowing the magnitude of the drawbar pull deflection, the transducer natural frequency is calculated to be $25.83 Hz$.

The transmissibility or magnification factor of the transducer is defined as the ratio of the transmitted force to the tractor and the dynamic force applied at tractor drawbar point. The transmissibility function for a spring-mass- damper system with a single degree of freedom under steady state is expressed as:

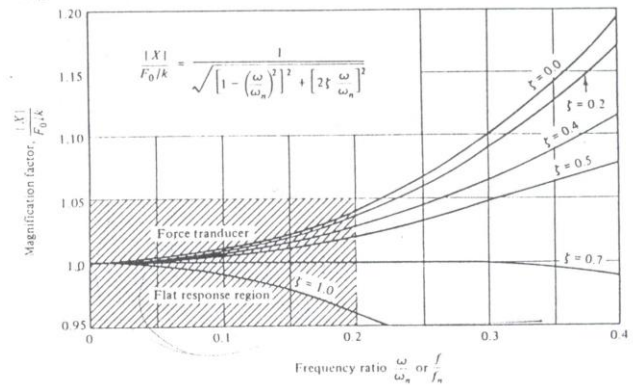
$$\frac{|X|}{F_0/k} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_t}{\omega_s}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega_s}{\omega_t}\right)\right]^2}} \quad (27)$$

The plot of Equation (27) in Fig.5 reveals that transmissibility is closed to unity for any values of frequency ratio ω_t / ω_n or f_t / f_n that is less than 0.1 regardless of the magnitude of any damping ratio or ξ . The working frequency of a tractor operating in the field is generally around $2 Hz$ [13 & 14]. The damping ratio for a drawbar transducer is assumed to be viscous in nature with magnitude equal to 0.01. Having both the damping ratio and frequency ratio (i.e .ratio of tractor working frequency to drawbar transducer natural frequency ($2/25.83$) being less than 0.1, gives a transmissibility magnitude of less than 1.01. In other words, this indicates that under dynamic response, the transducer would give a measurement error of no more than 1% of the excited fluctuating horizontal force. Such distortion or attenuation can be considered small in magnitude and significantly justified the accuracy of the transducer for dynamic response measurement.

2.4 Strain Gauges Installation

Installation of the strain gauges was carried out after smoothing the thin proof ring surfaces with fine silicon

carbide abrasive paper. The strain gauges were mounted accurately using epoxy adhesives at the 90-degree strain nodes on the inner and outer curve surfaces of the thin proof ring . SG280 protective coating was applied to protect the gauges against water, humidity and mechanical abrasion .The strain gauge circuitry is excited by a 5 Volts source and the output strain is recorded by the data acquisition in mVolt. The constan source is selected for better improvements in measurement linearity and accuracy .



Source :James *et al.* [15]

Figure (5). Dynamic response of force input at flat region

2.6 Calibration

Static calibration tests were carried out to determine the measurement linearity between applied load and output volt and to rectify the measurement accuracy between applied load and measured load. A Campbell Scientific CR1000 data acquisition system running on PC400W software was used to scan, and record output volt from the transducer. The calibration tests were performed by subjecting known loads to the drawbar pull transducer and recording the respective measured output volt via the data acquisition system (see Fig. 6). The drawbar transducer was mounted on to a universal loading frame that is equipped with a manual operated hydraulic power pack. A load cell is located in between the transducer and hydraulic system to measure the magnitude of the applied load by the hydraulic cylinder of the power pack. The data acquisition system was programmed to scan and record the measured signal of the drawbar pull transducer using Short cut or CR Basic program. The test was conducted under the loading range from 2 to 20 kN and unloading range from 20 to 2 kN at 2 kN intervals.

3. RESULTS AND DISCUSSION

3.1 Development of Drawbar Pull Transducer

A drawbar pull transducer had been successfully design and developed. The design of the developed drawbar pull transducer is based on a thin proof ring that forms an integral part of the drawbar element (see Fig. 7). This machined transducer element replaces the existing drawbar of the tractor without affecting its original towing point. The drawbar element is of hitch pin type having one end fixed to the tractor body using two pins and other end free for hitching the

implement using a single pin. The thin proof ring portion was made closed to the fixed end of the drawbar element to reduce lateral and longitudinal moments effect on the measurements. Mild steel, being low in modulus of elasticity, is employed in making the drawbar element to give greater strain sensitivity. The developed drawbar pull have overall dimensions 850 mm length and 90 mm width and 30 mm thickness while the thin ring portion have 35mm mean radius, 30mm width and 10 mm thickness.

3.2 Calibration

The plotted calibration graph in Fig. 8 shows applied load and measured output volt were highly correlated. The linearity equation is expressed by:

$$Y = 0.0034X + 0.1606 \quad \text{with} \quad R^2 = 0.9968 \quad (28)$$

where Y represented the measured output volt (mV) and X is the applied load in kN. The equation was used in the programming to scale Data logger (i.e multiplier from 1 to 294.12 and an offset of 0 to -47.24) to read the measured output volt from the tractor's drawbar pull in kN.

The measured strain gauge bridge sensitivity was 0.0034 mV/kN. This value was 33.24 times lower than the earlier computed theoretical sensitivity (i.e 0.113mV/kN). Again, the difference was due to gain multiplier effect that was set automatically during auto ranging by the Data logger.

The plotted measurement accuracy graph in Figure 9 shows high degree of linearity between applied load and measured load. Their relationship is best expressed by the following formula:

$$L_m = 1.0067L_a \quad \text{with} \quad R^2 = 0.9999 \quad (29)$$

where L_m represented the measured output load in kN and L_a is the applied load in kN.



Figure (7). Calibration setup of drawbar transducer

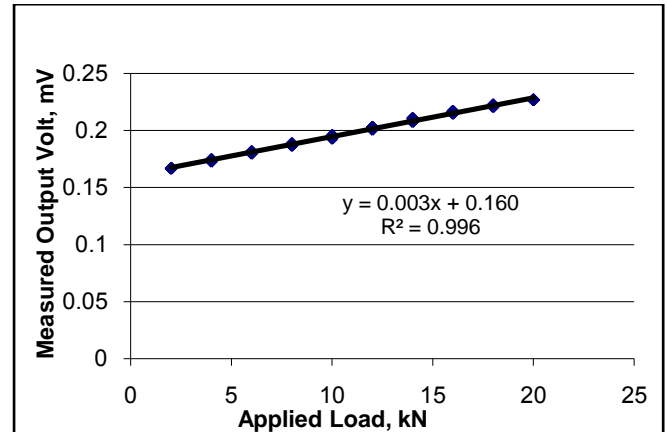


Figure (8). Calibration curve for Drawbar Pull Transducer

The transducer is rated to give measurement accuracy within $\pm 0.67\%$ range. This factor was used for computing and documenting the measured output load by the transducer. The calculated drawbar transducer deflection by strain energy theory was 0.38mm for a maximum design load of 20kN. The estimated drawbar transducer natural frequency from a single DOF of spring-mass-damper system was 25.83 Hz. A tractor-implement system with a harmonic frequency of 2Hz has a frequency ratio less than 0.1. The damping inherent is viscous in nature and can be assumed having a damping ratio or ξ equal to 0.01. With such a damping ratio and frequency ratio for the drawbar transducer, the transmissibility will be less than 1.01 which in turn results with a measured horizontal force error of no more than 1% of the excitation horizontal force. Consequently, the conducted static calibration of the transducer was acceptable for dynamic force measurement and ensured that the cyclic forces measured by transducer on the tractor would not be distorted or attenuated.

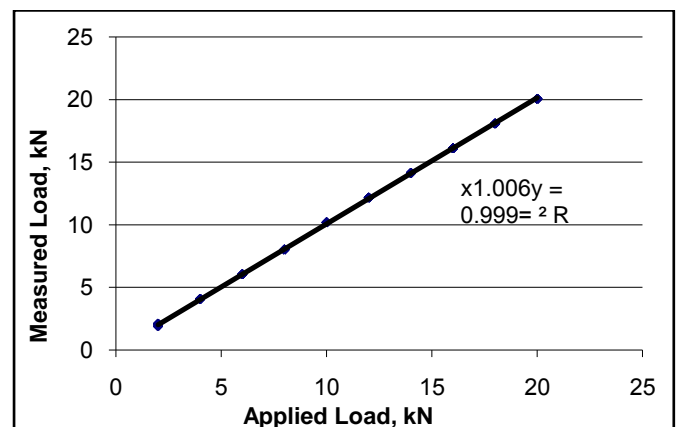


Figure (9). Verification curve for measurements accuracy of drawbar pull transducer

4. CONCLUSIONS

The following conclusions could be drawn from the obtained results:

- A drawbar pull transducer had been successfully designed, developed and calibrated to measure horizontal force up to 20 kN. The transducer design was based on a thin proof ring that forms an integral

part of the tractor drawbar. The machined drawbar element serves as a force transducer and at same time replaces the existing tractor drawbar without affecting its original towing point position.

- The transducer has been designed with good stiffness as indicated by its small deflection at the design load and excellence measurement sensitivity.
- The data acquisition system was successfully able to scan and record the measured signals by the drawbar pull transducer as programmed. The transducer was successfully able to scan and record by data acquisition system.
- Static calibration results showed excellent measurement linearity between applied load and output volt with coefficient of correlation or R^2 equal to 0.9968.
- The transducer measurements accuracy are within the acceptable range limits with measurement errors no more than 0.67% of the measured force magnitude under static measurements.

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