



Experimental evaluation of transmission length equations for pre-tensioned hollow core slabs

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Abstract: This paper presents an experimental evaluation of transmission length in hollow core slabs against 18 suggested equations in some codes of practice in addition to some equations from the previous literature. The experimental transmission length was predicted at 95% average mean strain (95% AMS) by measuring changes in strain after wire cutting on the near concrete face using both electric strain gauge and demountable mechanical strain gauges (DEMEC). The ratio between the measured transmission length and the value predicted using codes of practice equations varied from 1.0 to 1.26. On the other hand, the variation was 0.59 to 1.97 for the equations from the previous literature. The experimental results also confirmed the previous findings of non-linear prestress transfer over the transmission zone.

Keywords: Transmission length, transfer length, development length, Pre-tensioned concrete, bond.

1 Introduction

In pre-tensioned concrete systems, the strands are tensioned within long prestressing beds before the concrete hardening, this system allows the mass production of multiple smaller members at any desired length by cutting the long extended concrete cast for long distances [1]. During production the strands expanded and re-anchor themselves through certain length deeper in the concrete, eventually, the re-anchorage of the strands through elastic radial expansion become known as the Hoyer effect, and the length needed to re-anchor the strands was named development length (or the bond length) [1]. Whereas, The transmission length (L_T), is defined as the bonded length that is required to develop an effective prestressing force in a strand within the end anchorage zone [2] as shown in Fig .1.

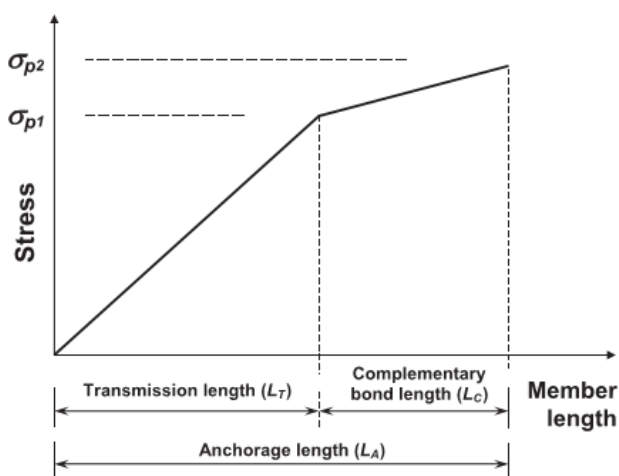


Fig .1. Idealization of the pre-stressing strand stress on the concrete surface along the strand from the cutting end [3].

Several theoretical and experimental works have been conducted on the transmission length of pre-stressing tendons over years to study the different parameters affecting the prestress transfer in pre-tensioned concrete [2]–[8] and post-tensioned concrete [9]–

[12]. Currently, several codes and studies generally accept the hypothesis of uniform bond stress distribution, which assumes linear variations of the pre-stressing reinforcement stress for both the transmission and complementary bond lengths, resulting in a bilinear model [3]. However, the previous studies show that the relation between the stresses in the tendon along the length is not based on linear relationships and influenced by many parameters [4]. This study aims to experimentally evaluate current formulae in codes and literature for predicting the transmission length in pre-tensioned hollow core concrete slabs.

2 Background

2.1 Bond mechanism

The transfer of stresses from prestressing steel to the surrounding concrete occurs as a result of bond between the two materials. The bond is essential for the transfer of forces between the strands and the concrete, without bond each material act independently. The mechanisms by which the concrete and prestressing steel bond together in pre-tensioned concrete are: adhesion; Hoyer's effect; and mechanical interlock [13]. As high and ultra-high concrete strength have been introduced in precast concrete usage a higher force has to be transmitted between steel and concrete, as a result, an increased attention should be paid to the prestress transfer and possible influencing parameters [1].

2.2 Development length

When a pre-tensioned concrete member is loaded, a complementary bond length beyond the transmission length is required to develop the ultimate pre-stress. This additional length from that is required to reach a design stress is known as the development length (or the anchorage length) [3]. The development length (L_d) is the sum of the transmission length (L_t) and the complementary bond length (L_b).

2.3 Importance of the transmission length

The importance of the transmission length became a necessity in the design process of the pre-tensioned concrete, it is required to

calculate shear and tensile stresses at the end anchorage zone of the structural member, as well as calculating the design capacity of a member [14]. According to the Euro code 2, the shear strength is a direct function of the transmission length [14], [15].

2.4 History of the transmission length formulae

The first introduction for the transmission length equation was in the American code of design ACI 318-63 [16] and it was adopted by AASHTO in 1973. The Portland Cement Association (PCA) first conducted the original research that formed the basis for the derivation of the transmission length equation in the late 1950s and early 1960s. However, the research reports did not formulate an equation to calculate the transmission length [3]. The ACI Committee 423 derived an equation, Equation 2 in Table 1, based on reappraisal of the PCA results to provide a reasonable mean for the data points rather than a conservative estimate [17]. In the ACI 318 [16], [18], AASHTO [19], [20], the transfer length is calculated (in units of MPa and mm) as shown in Equation 1 and 2 (in Table 1).

The European practice [15], [20] calculated the transfer length as shown in Equation 3 (in units of MPa and mm). Equation 3 considers the effect of the surrounding concrete tensile strength, (f_{ctd}), strand geometry, releasing method, and bond between prestressing steel and concrete.

Table 1 summarizes 18 empirical and analytical equations suggested for estimating the transmission length in pre-tensioned concrete structural elements in some codes and literature. For example, in ACI 318 [16], [18], the estimate of transmission length only includes strand diameter and effective prestress while in Eurocode 2 and Model Code [15], [21] account for concrete properties, strand type, release method, and bond condition [3].

All equation provided in Table 1 are empirical and assume linear prestress distribution in the transmission zone from zero up to the effective prestress except Equation 18 which is formulated using analytical model based on the thick-walled cylinder theory and considers linear material properties for both steel and concrete and gives exponential prestress distribution.

Table 1. Equations for the transmission length in some codes and previous literature [3].

Reference	Transmission length	Equation No.	Remarks
ACI 318 (1963-2008) [16], [18]	$L_t = \frac{\sigma_{psc} \cdot \phi}{20.7}$	1	ϕ = The nominal diameter of pre-stressing strand. σ_{psc} = The effective stress in pre-stressing strand after all prestress losses.
AASHTO (1973) [19]	$L_t = \frac{\sigma_{pi} \cdot \phi}{20.7}$	2	σ_{pi} = The effective stress in pre-stressing strand just after prestress transfer.
AASHTO LFRD (2004) [20]	$L_t = 60 \cdot \phi$	3	
Eurocode 2 (2004) [15]	$L_t = \alpha_1 \alpha_2 \phi \frac{\sigma_{pi}}{\eta_{p1} \eta_{p2} f_{ctdi}}$	4	α_1 = type of release, α_2 = area factor, η_{p1} and η_{p2} = account for the tendon type, η_1 = account the bond conditions, f_{ctd} and f_{ctdi} = concrete tensile strength.

Model code (2010) [21]	$L_t = \alpha_1 \alpha_2 \alpha_3 \frac{A_p}{\pi \phi} \frac{\sigma_{pi}}{\eta_{p1} \eta_{p2} f_{ci}}$	5	α_{p1} = type of release, α_{p2} = action affect to be verified, α_{p3} = bond situation, $\frac{A_p}{\pi \phi} = \frac{7}{36} \phi$, η_{p1} = account for the tendon type, η_{p2} = account the bond conditions, f_{ctd} and f_{ctdi} = concrete tensile strength
Shahawy et al. (1992) [22], [23]	$L_t = \frac{\sigma_{pi} \cdot \phi}{20.7}$	6	K = 1 slabs and slender members, K= 0.5 when La/h ≤ 3 (h = overall thickness of member)
Martin and scott (1967) [24]	$L_t = 80 \cdot \phi$	7	$L_A = \frac{\phi}{2.69} (\sigma_{pa} - \frac{1159}{\phi^{\frac{1}{6}}})$
Cousin et al. (1990) [25]	$L_t = \frac{\sigma_{psc} \cdot A_p}{\pi \phi U'_t \sqrt{f_{ci}}} + 0.5 \frac{U'_t \sqrt{f_{ci}}}{B}$	8	For uncoated strands: $U'_t =$ plastic transfer bond stress 0.556, $B =$ bond modulus = 0.0815 MPa/mm,
Mohamoud et al. (1992) [26]	$L_t = \frac{\sigma_{pt} \cdot \phi}{\alpha_t \cdot f_{ci}^{0.67}}$	9	$\alpha_t = 2.4$ for steel strands α_f no reported, σ_{psc} = effective prestress at loading
Deatherage et al. (1994) [27]	$L_t = \frac{\sigma_{pi} \cdot \phi}{20.7}$	10	
Buckner (1995) [28]	$L_t = \frac{\sigma_{pi} \cdot \phi}{20.7}$	11	
Mitchel et al. (1993) [29]	$L_t = \frac{\sigma_{pi} \cdot \phi}{20.7} \sqrt{\frac{20.7}{f_{ci}}}$	12	
Tadros and Baishy (1996) [30]	$L_t = \frac{(\sigma_{psc}/0.8) \cdot \phi}{20.7}$	13	
Lane (1998) [31]	$L_t = 4 \frac{\sigma_{pt} \cdot \phi}{f_i} - 127$	14	
Zia and mostafa (1977) [32]	$L_t = a \frac{\sigma_{pi} \cdot \phi}{f_i} - b$	15	For gradual release: $a = 1.3$; $b = 58$, For sudden release: $a = 1.5$; $b = 117$.
Kose and Burkett (2005) [33]	$L_t = 0.05 \frac{\sigma_{pt} \cdot (1 - \phi)^2}{\sqrt{f_{ci}}}$	16	
Marti et al. (2014) [3]	$L_t = \frac{\sigma_{pi} \cdot A_p}{C_p \cdot f_{ci}^{0.67}}$	17	
Abdelatif et al. (2015) [4]	$L_t = \frac{\phi}{4\mu} \left[\left(\frac{1}{B} + \frac{v_p}{B^2 E_s} \right) \cdot \ln \left(1 + 0.95 \frac{B}{A} \sigma_{psc} \right) - 0.95 \sigma_{psc} \left(\frac{1 - v_p}{E_p} + \frac{v_p}{B E_p} \right) \right]$	18	μ = Friction coefficient, v_p = passion ratio, E_c and E_p = Modulus elasticity for concrete and steel, respectively. $A=B=0.088$ for $f_{ci} = 35$ MPa, $v_p = 0.2$, $E_p = 200$ MPa, $E_c = 20 + 0.2 f_{ci}$.

3 Experimental work

3.1 Properties of the Slab

To fulfill the main aim of the study, the formulae in Table 1 are subjected to experimental evaluation using measured transmission length in a pre-tensioned hollow core slab in this paper. The slab used in this test has properties shown in Table 2.

Table 2. Properties of the pre-tensioned hollow core slab

Property	Value
Slab total length	3.89 m (150 in)
Slab width	0.89 m (35 in)
Slab depth	0.15 m (6 in)
Strand nominal diameter	7 mm (0.27 in)
Initial stress	1165 MPa (169 Ksi)
Designed concrete compressive strength	35 N/mm ²

3.2 Test procedure

An experimental program was conducted to measure the transmission length for 7 mm wire in locally manufactured pre-stressed hollow core slabs. The transmission length was calculated using 95% average maximum strain (95% AMS) method [4]. In this method the transmission length is estimated at the intersection of the strain on the concrete surface with 95% of the average maximum strain in the pre-stressed tendons [13]. The strains were measured at concrete surface along the length and parallel to the prestressing wire. At each test the strains are measured before and after the prestress is released after cutting through the slab length. The strain was measured using both mechanical and electrical strain gauges. The used electrical resistance strain gauges were 30 mm long type FL-30-11 produced by the TML Company, Japan. Moreover, as backup, the demountable mechanical strain gauges (DEMEC) are used by measuring the distance between the pins before and after cutting of the strands. The strain was recorded in interval time of 5 min, 15 min, 30 min, 45 min, 1 hour, 1.5 hour and 3 days to show the effect of time in the early age after cutting, utilizing TML TDS 303 data logger as shown in Fig .2. The cutting distances was chosen to obtain a consistent strain profile in the expected transmission zone as well as giving at least two readings in a region of constant strain beyond the end of the transmission zone. The slab was instrumented on the concrete face near to the prestressing steel wire which is cut using concrete core machine as shown in Fig .3.

4 Measurements of the transmission length

Fig .4 and Fig .5 give the measurements of strain changes over the time after cutting the wire using electrical strain gauges and DEMEC, respectively. The results show that the strain profile changes over the time after cutting and finally stabilized when the prestress transfer from steel to concrete completed. The value of the transmission length using 95% AMS method is predicted as 350 mm from the cutting end. Both methods of measuring strain are found to be reliable and confirmed non-linear prestress distribution over the transmission zone.

5 Comparison between experimental results and equations in literature

In this study, the effective pre-stress was considered after losses as 84% of the initial pre-stress value. The tensile strength was calculated as a tenth of concrete compressive strength for the Eurocode2 (EC2) [15] and the Model Code [21].

It should be noted that, Equations in Table 1 have variations in terms used for initial and effective pre-stress.

5.1 Comparison between experimental results and equations in some codes of practice

The measured value of the transmission length from the experimental work was compared with the some code of practice, namely AASHTO, ACI 318, Model Code, EC2, and AASHTO LRFD in Fig .6.

For the EC2, the cylindrical compressive strength was calculated as ($f_{ci}=0.8f_c$), and the constant values were considered for a gradual release ($\alpha_1 = 1.0$), seven wire strand ($\alpha_2 = 0.19$), and good bond concrete ($\eta_1 = 1.0$). Where ($\alpha_1 = 1.0$) for gradual pre-stress force release or ($\alpha_1 = 1.25$) for sudden release, and ($\alpha_2 = 0.25$) for plain tendons or ($\alpha_1 = 0.19$) for seven wire strand, ($\eta_p = 2.7$) for indented wires or ($\eta_p = 3.2$) for seven-wire strand, ($\eta_1 = 1.0$) for good bond condition and ($\eta_1 = 0.7$) otherwise. For the Model Code similar conditions to the EC2 were used to calculate the constants values. Moreover, considering the effect of moment and shear capacity on the design ($\alpha_{p2} = 1.0$), ($\alpha_{p2} = 0.5$) for verification of transverse stress in anchorage zone, a value of ($\alpha_{p2} = 1.0$) has been adopted.

The measured value of the transmission length from the experimental work was compared with empirical equation in some codes of practice as shown in Fig .6.

Fig .6 shows that the prediction of both AASHTO and ACI 318 were very close to the measured transmission length while the Model Code, EC2, and AASHTO LRFD overestimate the transmission length by +11.8%, +11.8% and +26.3%, respectively.

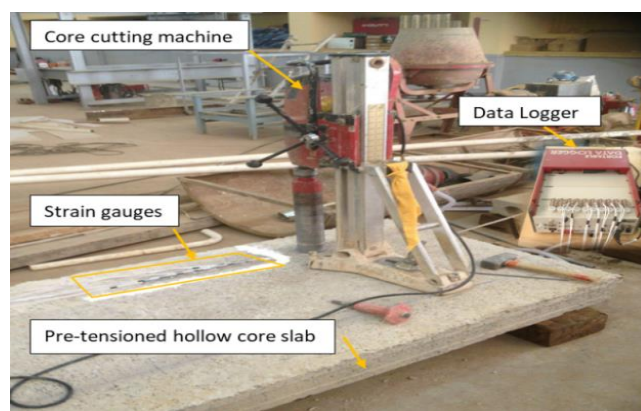


Fig .2. Experimental setup to measure strain for prediction of transmission length in hollow core slab

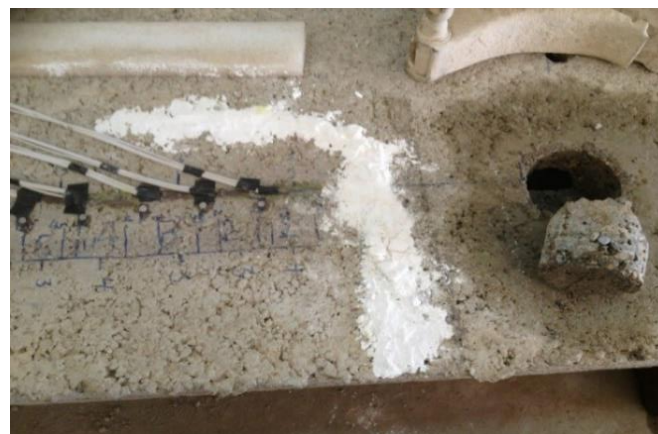


Fig .3. Instrumentation of strain gauges along the transmission zone and cutting point

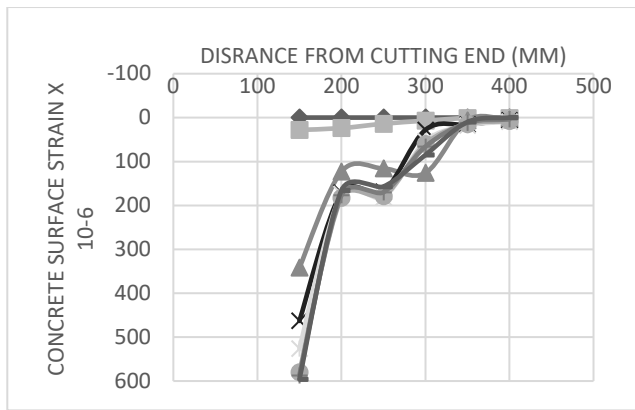


Fig .4. Measured strain on the concrete surface along the wire from the cutting point using and electrical resistance strain gauge (ERSG).

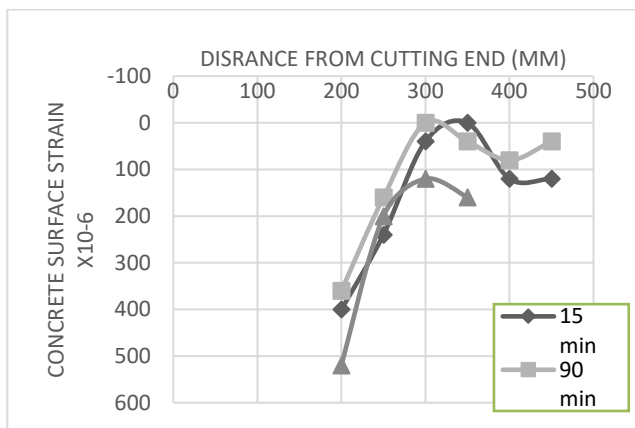


Fig .5. Measured strain on the concrete surface along the wire from the cutting point using the demountable mechanical strain gauge (DEMEC).

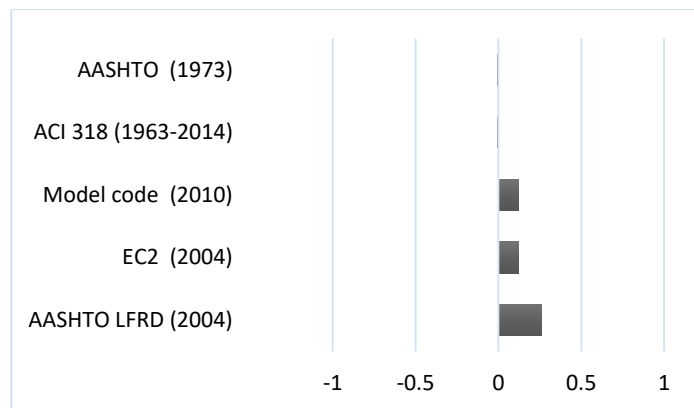


Fig .6. The ratio of difference between the measured transmission length (L_t) and the calculated values from the equations in codes of practice.

5.2 Comparison between experimental results and equations in previous literature

In Fig .7, the experimental value is compared to 13 equations of transmission length. The results are organized in four groups: 1) those which underestimate the transmission length by -20.7%, -23.5%, and -40.9% (i.e. 0.59 L_t), 2) those which are much closer, 3) those which slightly overestimate the results by +6.7%, and 4) those which overestimate the transmission length by +24.4%, +57.8%, +68.4%, and +97.3%. Based on the results only equations from group 2 and 3 gave a good agreement.

For example Equation 15 from the work of Zia and Mostafa (1997) [32] resulted in lower estimation of the transmission

length, because it was based on an analysis of various experimental works. Also, the prediction from Mahmoud et al. (1992) [26] and Mitchel et al. (1993) [29], Equation 9 and Equation 11 give shorter transmission length because they were derived for the transmission length in the concrete with high strength or the high yield tendons. The comparison also shows that using prediction of Lane (1998) [31] which is based on transmission length for greater diameters results in longer transmission length. Prediction of Abdelatif et. al. [4], Equation 18, which depends on the materials properties gives only 6.7% higher from the measured transmission length in the laboratory.

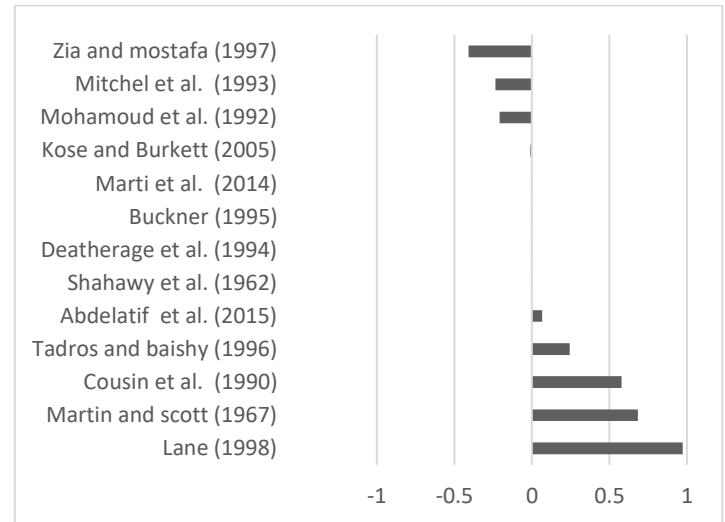


Fig .7. The ratio of difference between the measured transmission length (L_t) and the calculated values from the equations in previous studies

6 Conclusions

This paper presented an evaluation of 18 equations suggested for prediction of transmission length in some codes of practice and previous literature against experimental measurements on a pre-tensioned hollow core slab conducted in this research. The experimental results confirmed the non-linear distribution of prestress over the transmission zone, however 17 out of 18 of the presented equations in codes and literature suggest linear distribution. The prediction of AASHTO and ACI 318 codes were found to be very close to the measured transmission length. On the other hand the Model Code, EC2, and AASHTO LRFD was found to overestimate the transmission length. This also was the case for the equations in literature, some give closed results while the rest either overestimate or underestimate the transmission length. The reason for this discrepancy is that some equations are based on experimental work on pre-tensioned concrete of a limited range of variation in the properties and element type.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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