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Design of a Robust Decentralized Power System Stabilizer for Sudan National Grid

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Abstract: Power system stabilizers (PSS) are used to generate supplementary control signals for the excitation system in order to damp the low frequency, generator angle, speed variation, generator power, voltage magnitude and power flow oscillations. This paper presents robust decentralized power system stabilizer (PSS) design approaches for a power system consisting of 41 machines with 167 buses. This paper mainly focuses on developing robust decentralized control techniques for power systems, with special emphasis on problems that can be expressed as minimizing a linear objective function under linear matrix inequalities [LMI] in tandem with bilinear matrix inequalities [BMI] constraints. The design problem is considered the natural extension of the reduced order for decentralized dynamic output H2/H∞ -norm controller's synthesis for power systems. The resulting optimization problem has a general bilinear matrix inequalities (BMIs) form which can be solved using an iterative linear matrix inequalities [LMIs] programming method. Simulations were carried out using loss of line without fault tests at transmission line on Sudan grid.

Keywords: *Decentralized control; power system stabilizers; robust control; nonlinear systems.*

1. INTRODUCTION

Most of the generating units recently added to the Sudanese grid were equipped with continuously-acting voltage regulators. As this unit constitutes a large percentage of the generating capacity, it became apparent that the voltage regulator action had a detrimental impact upon the dynamic stability of the power system.

The deregulation of the electricity market has led to increasing uncertainties concerning power flow within the network. Oscillations of small magnitude and low frequency often persisted for long periods of time and in some cases presented limitations on power transfer capability. This is further compounded by the physical expansion of interconnected networks, which makes the prediction of system response to disturbances and severe loading condition more difficult [1].

Power system stabilizers were developed to aid in damping these oscillations via modulation of the generator excitation [1], [2]. These and other similar developments prompted both power and control engineers to use new controller design techniques and more accurate model descriptions for the power system components with the objective of providing reliable electricity services. To meet modern power system requirements, controllers have to guarantee robustness over a wide range of system operating conditions, and this further highlights the fact that robustness is one of the major issues in power system controller design. Recently, a number of efforts have been made to extend the application of robust control techniques to power systems, such as L_∞ optimization [1, 2], H_∞- optimization and structured singular value (SSV or μ) technique [3-5].Tuning of supplementary excitation controls for stabilizing system modes of oscillation has been the subject of much research during the past 10 to 15 years. The secure operation of power systems requires the application of robust controllers, such as Power System Stabilizers (PSS), to provide sufficient damping at all credible operating conditions [1].

This paper focuses on the extension of linear matrix inequalities (LMIs) based mixed H_2/H_∞ optimization approach to problems of practical interest in power systems. The design problem considered is the natural extension of the reduced order decentralized dynamic output H_2/H_∞ controller's synthesis for power systems. In the design, the fixed-structure H_2/H_∞ dynamic decentralized output feedback controller problem is first reformulated as an extension of static output feedback controller design problem for the extended system. The resulting optimization problem has a general bilinear matrix inequalities (BMIs) form which can be solved using iterative LMIs programming method based on linearizing the objective function with respect to its variables. Moreover, the paper also presents a general approach that can be used for designing any order robust PSS structure controllers in power system. The application of this approach to a multi-machine power system allows a coordinated tuning of controllers that incorporate robustness to changes in the operating conditions as well as model uncertainties in the system. Therefore, this paper proposes alternative computational schemes that solve the robust decentralized controllers design problem for power systems using [6]:

- Sequential linear matrix inequality programming.
- Generalized parameter continuation method involving matrix inequalities.

2. OUTLINE OF THE PROBLEM

- **System Model**

Consider the general structure of the i^{th} – generator together with the PSS block in a multi-machine power system shown in Figure 1. The input of the i^{th} - controller is connected to the output of the washout stage filter, which prevents the controller from acting on the system during steady state. Let the structure of this i^{th} - washout stage be given by [6]:

$$\Delta y(s) = \frac{sT_{Wi}}{1+sT_{Wi}}\Delta\omega(s) \quad (1)$$

To illustrate the design procedure, consider the following first-order PSS controller with *a-priori* assumption made on the value of T_{i2}:

$$K_i\left[\frac{1+sT_{i1}}{1+sT_{i2}}\right] \quad (2)$$

The PSS structure in eq. (2) can be further rewritten in the following form

$$K_i\left[\frac{1+sT_{i1}}{1+sT_{i2}}\right] = \left[K_{i1} + K_{i2}\frac{1}{1+sT_{i2}}\right] \quad (3)$$

where K_{i1} and K_{i2} are easily identified as gain parameters that are to be determined during the design. Moreover, the gain parameters K_{i1} and K_{i2} together with T_{i2} determine the original parameters K_i and T_{i1}.

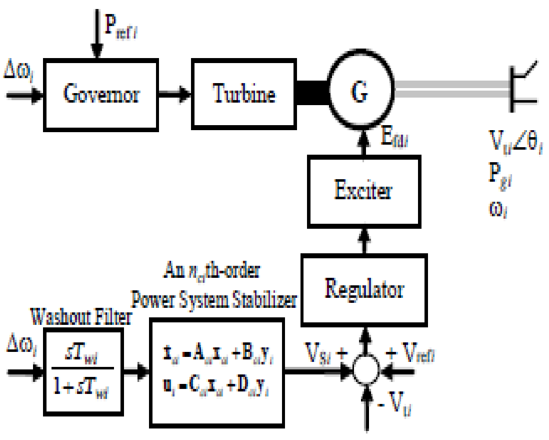


Fig. 1. General structure of the i^{th} - generator together with the PSS structure and washout stage in a multi-machine power system

After augmenting the washout stage in the system, the i^{th} - subsystem, within the framework of H_2/H_∞ design, is described by the following state space equation:

$$\begin{aligned} \dot{x} &= A_{ii}x_i + \sum_{j\neq 1} A_{ij}x_j + B_{i0}W_{i0} + B_{i1}W_{i1} + B_{i2}u_i \\ z_i &= C_{i1}x_i + D_{i10}w_{i0} + D_{i11}w_{i1} + D_{i12}u_i \\ y_i &= C_{iy}x_i + D_{iy0}w_{i0} + D_{iy1}u_{i1} \end{aligned} \quad (4)$$

Where $x_i \epsilon \mathcal{R}^{n_i}$ is the state variable, $u_i \epsilon \mathcal{R}^{n_i}$ is the control input, $y_i \epsilon \mathcal{R}^{n_i}$ is the measurement signal, $z_i \epsilon R^{nz_i}$ is the regulated variables, $w_{i0} \epsilon R^{nw_{i0}}$ and $w_{i1} \epsilon R^{nw_{i1}}$ are exogenous signals (assuming that $\mathbf{w}_{i1}$ is either independent of $\mathbf{w}_{i0}$ or dependent causally on $\mathbf{w}_{i0}$ for the i^{th} - subsystem [7].

Now consider the following approach to design decentralized robust optimal H_2/H_∞ controllers of the form eq. (3) for the system given in eq. (4), i.e. determining optimally the gains K_{i1} and K_{i2} within the framework of H_2/H_∞ optimization. This implies the incorporation of the dynamic part of the controller in eq. (4), namely

$$\begin{bmatrix} 1 & {}^{1}/_{sT_{i2}} \end{bmatrix}^T \quad (5)$$

and then reformulating the problem as an extension of a static output feedback problem for the extended system. Hence, the state space equation for i^{th} – subsystem becomes:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x} \end{bmatrix} &= \begin{bmatrix} A_{ii} & 0 \\ B_{ci}C_{iy} & A_{ci} \end{bmatrix}\begin{bmatrix} x_i \\ x_{ci} \end{bmatrix} + \begin{bmatrix} \sum_{j\neq i} A_{ij}x_j \\ 0 \end{bmatrix} + \begin{bmatrix} B_{i0} \\ B_{ci}D_{iy0} \end{bmatrix} w_{i0} + \begin{bmatrix} B_{i1} \\ B_{ci}D_{iy1} \end{bmatrix} w_{i1} + \begin{bmatrix} B_{i2} \\ 0 \end{bmatrix} u_i \\ z_i &= \begin{bmatrix} C_{i1} & 0 \end{bmatrix}\begin{bmatrix} x_i \\ x_{ci} \end{bmatrix} + D_{i10}w_{i0} + D_{i11}w_{i1} + D_{i12}u_i \\ \tilde{y} &= \begin{bmatrix} D_{ci}C_{iy} & C_{ci} \end{bmatrix}\begin{bmatrix} x_i \\ x_{ci} \end{bmatrix} + D_{ci}D_{iy0}w_{i0} + D_{ci}D_{iy1}w_{i1} \end{aligned} \quad (6)$$

WhereA_{ci},B_{ci}, C_{ci} and D_{ci} are the state space realization of eq. (5) and are given by:

$$A_{ci} = \left[-{}^{1}/_{T_{i2}}\right], B_{ci} = \left[{}^{1}/_{T_{i2}}\right], C_{ci} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D_{ci} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (7)$$

Finally, the overall extended system equation for the system can be rewritten in one state space model as

$$\begin{aligned} \tilde{x} &= \tilde{A}\tilde{x} + \tilde{B}_0w_0 + \tilde{B}_1w_1 + \tilde{B}_2u \\ z &= \tilde{C}_1x + \tilde{D}_{10}w_0 + \tilde{D}_{11}w_1 + \tilde{D}_{12}u \\ \tilde{y} &= \tilde{C}_yx + \tilde{D}_{y0}w_0 + \tilde{D}_{y1}w_1 + \tilde{D}_{y2}u \end{aligned} \quad (8)$$

Where

$$\tilde{A}=\begin{bmatrix} A_{11} & 0 & A_{12} & 0 & A_{13} & 0\cdots & A_{1N} & 0 \\ B_{c1}C_{12} & A_{12} & 0 & 0 & 0 & 0\cdots & 0 & 0 \\ A_{21} & 0 & A_{22} & 0 & A_{23} & 0\cdots & A_{2N} & 0 \\ 0 & 0 & B_{c2}C_{22} & A_{c2} & 0 & 0\cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\ddots & \vdots & 0 \\ A_{N1} & 0 & A_{N2} & 0 & A_{N3} & 0\cdots & A_{NN} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0\cdots & B_{cN}C_{N2} & A_{cN} \end{bmatrix}$$

$$\tilde{B}_0 = \text{blkdiag}\left\{\begin{bmatrix} B_{10} \\ B_{c1}D_{1y0}\end{bmatrix}\begin{bmatrix} B_{20} \\ B_{c2}D_{2y0}\end{bmatrix}\cdots\begin{bmatrix} B_{N0} \\ B_{cN}D_{Ny0}\end{bmatrix}\right\}$$

$$\tilde{B}_1 = \text{blkdiag}\left\{\begin{bmatrix} B_{11} \\ B_{c1}D_{1y1}\end{bmatrix}\begin{bmatrix} B_{21} \\ B_{c2}D_{2y1}\end{bmatrix}\cdots\begin{bmatrix} B_{N1} \\ B_{cN}D_{Ny1}\end{bmatrix}\right\}$$

$$\tilde{B}_2 = \text{blkdiag}\left\{\begin{bmatrix} B_{12} \\ 0\end{bmatrix}\begin{bmatrix} B_{22} \\ 0\end{bmatrix}\cdots\begin{bmatrix} B_{N2} \\ 0\end{bmatrix}\right\}$$

$$\tilde{C}_1 = \text{blkdiag}\{[C_{11} \quad 0][C_{21} \quad 0]\ldots[C_{N1} \quad 0]\}$$

$$\tilde{C}_2 = \text{blkdiag}\{[D_{c1}C_{1y} \quad C_{c1}][D_{c2}C_{2y} \quad C_{c2}]\ldots[D_{cN}C_{Ny} \quad C_{cN}]\}$$

$$\widetilde{D}_{10} = \text{blkdiag}\{D_{110}, D_{210}, \ldots D_{y10}\}$$

$$\widetilde{D}_{11} = \text{blkdiag}\{D_{111}, D_{211}, \ldots D_{y11}\}$$

$$\widetilde{D}_{12} = \text{blkdiag}\{D_{112}, D_{212}, \ldots D_{y12}\}$$

$$\widetilde{D}_{y0} = \text{blkdiag}\{D_{c1}D_{1y0}, D_{c2}D_{2y0}, \ldots D_{cN}D_{Ny0}\}$$

$$\widetilde{D}_{y1} = \text{blkdiag}\{D_{c1}D_{1y1}, D_{c2}D_{2y1}, \ldots D_{cN}D_{Ny1}\}$$

Hence, the static output feedback controller for ith - subsystem is given as:

$$u_i = \widetilde{K}_i\tilde{y}_i \tag{9}$$

Where $\widetilde{K}_i = [K_{i1} \quad K_{i2}]$ .Moreover, the decentralized static output feedback controller for the whole system will then have the familiar block structure of the form

$$u = \widetilde{K}_D\tilde{y} \tag{10}$$

Where $\widetilde{K}_D = \text{blkdiag}(\widetilde{K}_1, \widetilde{K}_2, \ldots\ldots\widetilde{K}_N)$. Substituting the static output feedback strategy (10) into the system equation of eq. (8), the closed loop system will become

$$\dot{\tilde{x}}=A_{ci}\tilde{x} + B_{ci0}w_0 + B_{ci1}w_1$$

$$z = C_{ci1}x + D_{ci0}w_0 + D_{ci2}w_1$$

Where,

$A_{cI} = \widetilde{A} + \widetilde{B}_2\widetilde{K}_D\widetilde{C}_y$, $B_{cI0} = \widetilde{B}_0 + \widetilde{B}_2\widetilde{K}_D\widetilde{D}_{y0}$, $B_{cI1} = \widetilde{B}_1 + \widetilde{B}_2\widetilde{K}_D\widetilde{D}_{y1}$,
$C_{cI1} = \widetilde{C}_1 + \widetilde{D}_{12}\widetilde{K}_D\widetilde{C}_y$,
$D_{cI0} = \widetilde{D}_{10} + \widetilde{D}_{12}\widetilde{K}_D\widetilde{D}_{y0}$,
$D_{cI1} = \widetilde{D}_{11} + \widetilde{D}_{12}\widetilde{K}_D\widetilde{D}$,

3. ROBUST DECENTRALIZED DYNAMIC CONTROLLER DESIGN

3.1 Mathematical Model for Large - Scale Systems

Consider a large-scale interconnected system S composed of N subsystems S_i, i =1, 2, … K, N described by the following equations:

$$\dot{x}_{i(t)} = A_ix_i(t) + B_iu_i(t) + \sum_{j=1}^{N}\left(M_{ij} + \Delta M_{ij}(t)\right)x_j(t)$$

$$y_i(t) = C_ix_i(t) \tag{11}$$

Where $x_i(t) \in \mathcal{R}^{n_i}$ is the state vector, $u_i(t) \in \mathcal{R}^{n_i}\,\Re$ is the control variable, $y_i(t) \in \mathcal{R}^{n_i}$ is the output variable of the subsystem S_i . The matrices A_i, B_i, $C_i$and $M_{ij}$are constant matrices of appropriate dimensions conformable to each S_i. Furthermore, the matrix M_{ij}represents the interconnections and/or interactions among the subsystems. The term Δ M_{ij} (.)is intentionally introduced to take into account the effect of any deviation from the given operating condition due to nonlinearities and structural changes in the system [8].

The interconnections and uncertainties terms in eq. (11), which are used to characterize the interactions among the subsystems and the effects of nonlinearities within each subsystem of a power system, can be rewritten in the following form.

$$h_i(t,x) = \textstyle\sum_{j=1}^{N}\left(M_{ij} + \Delta M_{ij}\right)x_j(t) \tag{12}$$

Furthermore, assume that the following quadratic constraints hold:

$$h_i(t,x)^Th_i(t,x) \leq \xi_i^2x^T(t)H_i^TH_ix(t) \tag{13}$$

Where ξi> 0 *are* parameters related to interconnection uncertainties in the system and H_i are matrices that reflect the nature of interconnections among subsystems. Moreover, assume that the pairs (A_i,B_i) and (A_i,C_i) are stabilizable and detectable, respectively. With the assumption of no overlapping among x_i (*t*), the state variable $x(t)\epsilon R^n$ of the overall system is denoted by:

$$x(t) = [x_1^T(\cdot), x_2^T(\cdot), \ldots\ldots x_N^T(\cdot)]^T$$

Thus, the interconnected systems can then be written in a compact form as [8]

$$\dot{x}(t) = A_Dx(t) + B_Du(t) + h(t,x)$$

$$y(t) = C_Du(t) \tag{14}$$

Where $x\,\epsilon R^q$is the state, $u \in R^m$is the input and $y \in R^q$is the output of the overall system S, and all matrices are constant matrices of appropriate dimensions with:

$A_D = \text{diag}\,\{A_1, A_2, \cdots, A_N\}$,
$B_D = \text{diag}\{B_1, B_2, \cdots, B_N\}$,
$C_D = \text{diag}\{C_1, C_2, \cdots, C_N\}$

The interconnection and uncertainty function $h(t,x) = [h_1^T(t,x), h_2^T(t,x), \cdots, h_N^T(t,x)]^T$ is bounded as

$$h^T(t,x)h(t,x) \leq x^T(t)\left[\sum_{J=1}^{N} \xi_i^2 H_i^T H_i\right]x(t) \quad (15)$$

In the following, consider designing a decentralized dynamic output feedback controller of an n_{ci} th- order for the i th-subsystem given in eq. (11) of the form [8]:

$$\dot{x}_{ci(t)} = A_{ci}x_i(t) + B_{ci}y_i(t)$$

$$u_i(t) = C_{ci}xc_i(t) + D_{ci}y_i(t) \quad (16)$$

Where $x_i(t) \in \mathcal{R}^{n_i}$ is the state vector of the ith-local controller and A_{ci}, B_{ci}, C_{ci} and D_{ci} are constant matrices to be determined during the design. After augmenting the i th controller in the system, the state space equation for the i th extended subsystem will have the following form

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{x}_{ci}(t) \end{bmatrix} = \begin{bmatrix} A_i + B_iD_{ci}C_i & B_iC_{ci} \\ B_{ci}C_i & A_{ci} \end{bmatrix}\begin{bmatrix} x_i(t) \\ x_{ci}(t) \end{bmatrix} + \begin{bmatrix} h_i(t,x) \\ 0 \end{bmatrix} \quad (17)$$

Which further can be rewritten as

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{x}_{ci}(t) \end{bmatrix}\left\{\begin{matrix}\begin{bmatrix} A_i & 0_{n_i\times n_{ci}} \\ 0_{n_{ci}\times n_i} & 0_{n_{ci}\times n_{ci}} \end{bmatrix}\begin{bmatrix} 0_{n_{ci}\times n_{ci}} & B_i \\ I_{n_{ci}} & 0_{n_i\times n_{ci}} \end{bmatrix}\begin{bmatrix} A_{ci} & B_{ci} \\ C_{ci} & D_{ci} \end{bmatrix} \\ \begin{bmatrix} 0_{n_{ci}\times n_{ci}} & I_{n_{ci}} \\ C_i & 0_{q_i\times n_{ci}} \end{bmatrix}\end{matrix}\right\}$$

$$\begin{bmatrix} x_i(t) \\ x_{ci}(t) \end{bmatrix} + \begin{bmatrix} h_i(t,x) \\ 0 \end{bmatrix} \quad (18)$$

With minor abuse of notation, the above equation can be rewritten in a closed form

$$\dot{\tilde{x}}_i(t) = \left[\tilde{A}_i + \tilde{B}_iK_i\tilde{C}_i\right]\tilde{x}_i(t) + \tilde{h}_i(t,\tilde{x}) \quad (19)$$

Where $\tilde{x}_i(t) = [x_i^T(t) \quad x_{ci}^T(t)]^T$, and the matrices $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{C}_i$ and K_i are given as follows

$$\tilde{A}_i = \begin{bmatrix} A_i & 0_{n_i\times n_{ci}} \\ 0_{n_{ci}\times n_i} & 0_{n_{ci}\times n_{ci}} \end{bmatrix} \tilde{B}_i = \begin{bmatrix} 0_{n_{ci}\times n_{ci}} & B_i \\ I_{n_{ci}} & 0_{n_{ci}\times m_i} \end{bmatrix} \tilde{C}_i = \begin{bmatrix} 0_{n_{ci}\times n_{ci}} & I_{n_{ci}} \\ C_i & 0_{q_i\times n_{ci}} \end{bmatrix}, K_i = \begin{bmatrix} A_{ci} & B_{ci} \\ C_{ci} & D_{ci} \end{bmatrix} \quad (20)$$

Moreover, the function $\tilde{h}_i(t,\tilde{x})$ satisfies the following quadratic constraint

$$\tilde{h}_i(t,\tilde{x})^T\tilde{h}_i(t,\tilde{x}) \leq \xi_i^2\tilde{x}^T(t)\widetilde{H}_i^T\widetilde{H}_i\tilde{x}(t) \quad (21)$$

Where the $\widetilde{H}_i \in \mathcal{R}^{pi\times(n+\sum_i^N n_{ci})}$ is partitioned according to the following

$$\widetilde{H}_i = \left[H_{i1} \quad 0_{n\times n_{c1}} \vdots H_{i2} \quad 0_{n\times n_{c2}} \vdots \cdots \vdots H_{iN} \quad 0_{n\times n_{cN}}\right] \quad (22)$$

Thus, the overall interconnected system can then be rewritten in a compact form as eq. (19)

$$\dot{\tilde{x}}(t) = \left[\tilde{A}_D + \tilde{B}_DK_D\tilde{C}_D\right]\tilde{x}(t) + \tilde{h}(t,\tilde{x}) \quad (23)$$

Where $\tilde{x}(t) = [\tilde{x}_1^T(t), \tilde{x}_2^T(t), \ldots\ldots, \tilde{x}_N^T(t)]^T$ and all matrices are constant matrices of appropriate dimensions i th

$$\tilde{A}_D = \text{diag}\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_N\},$$
$$\tilde{B}_1 = \text{diag}\{\tilde{B}_1, \tilde{B}_2, \cdots, \tilde{B}_N\},$$
$$\tilde{C}_D = \text{diag}\{\tilde{C}_1, \tilde{C}_2, \cdots, \tilde{C}_N\}.$$

Moreover, the function $h^T(t,x)h(t,x) \leq x^T(t)\left[\sum_{J=1}^{N} \xi_i^2 H_i^T H_i\right]x(t)$ bounded as

$$\tilde{h}^T(t,\tilde{x})h(t,\tilde{x}) \leq \tilde{x}^T(t)\left[\sum_{J=1}^{N} \xi_i^2 \widetilde{H}_i^T \widetilde{H}_i\right]x(t) \quad (24)$$

are instrumental in establishing the robust stability of the closed-loop interconnected system eq. (23) via a decentralized robust control strategy (16) under the constraints eq. (24) on the function $\tilde{h}(t,\tilde{x})$ [4].

Thus, the problem of designing a decentralized control strategy for the interconnected system (11), which at the same time maximizing the tolerable upper bounds on the interconnection and nonlinearity uncertainties, has the following form

$$\text{Min} \quad Trace\ (\Gamma) \quad (25)$$

Subject to (24)

Where $\Gamma = \text{diag}\ \{\gamma_1, \gamma 2, \ldots, \gamma_N\}$.

Therefore, the problem of designing a decentralized control strategy given in eq. (25) for the interconnected system eq. (11) can be restated as a non-convex optimization problem. The coupling constraint can be further relaxed as an LMI condition as follows:

$$\begin{bmatrix} Y_D & 1 \\ 1 & X_D \end{bmatrix} \geq 0 \quad (26)$$

Furthermore, using the cone-complementarity approach, there exists a decentralized robust output stabilizing controller K_D if the global minimum of the following optimization problem

$$\text{Min} \quad Trace\ (\Gamma) + Trace\ (Y_D, X_D) \text{Subject to } Y_D > 0\ , X_D > 0, \Gamma > 0 \quad (27)$$

Eq. (26) and (27) is $\alpha^* + \sum_{i=1}^{N} n_i$

Where $Trace\ (\Gamma) \leq \alpha^*$

3.2. Robust Decentralized Dynamic Output Feedback Controller Design Using Sequential LMI

– **Programming Method**

The optimization in eq. (27) is a non-convex optimization problem due to the bilinear matrix term in the objective functional. To compute the (sub)-optimal solution of this problem, an algorithm based on a sequential LMIs programming method is proposed. The idea behind this algorithm is to linearize the cost function in eq. (27) with respect to its variables and then to solve the resulting convex optimization problem subject to the LMI condition eq. (25) at each iteration. Moreover, the algorithm will set appropriately the direction of the feasible solution by solving a subclass problem of Newton-type updating coefficient. Furthermore, the solution of the optimization problem is monotonically non-increasing, i.e. the solution decreases in each iteration with the lower bound being $\sum_{i=1}^{N} n_i$ plus some positive number. Thus, the sequential LMI programming method for finding the decentralized dynamic output feedback controllers has the following optimization algorithms [9].

4. ROBUST DECENTRALIZED H∞ CONTROLLER DESIGN

4.1 Problem Formulation

Consider the general structure of the i^{th}-generator together with an nci^{th}-order PSS in a multi-machine power system shown in Fig. (1). The input of the i^{th}-controller is connected to the output of the washout stage filter, which prevents the controller from acting on the system during steady state. After augmenting the washout stage in the system, the i^{th} subsystem, within the framework of H_∞ design, is described by the following state space equation as taken in eqs. (24). [10]

$$\dot{x}_i(t) = A_{ii}x_i(t) + \sum_{j\neq 1} A_{ij}x_j(t) + B_{i0}W_{i0}(t) + B_{i1}W_{i1}(t) + B_{i2}u_i(t)$$

$$z_i(t) = C_{1i}x_i(t) + D_{11i}w_i(t) + D_{12i}u_i(t) \quad (28)$$

$$y_i(t) = C_{yi}x_i(t) + D_{y1i}w_i(t)$$

Where $x_i \epsilon \mathcal{R}^{n_i}$ is the state variable, $u_i \epsilon \mathcal{R}^{m_i}$ is the control input, $y_i \epsilon \mathcal{R}^{q_i}$ is the measurement signal, $z_i \epsilon \mathcal{R}^{p_i}$ is the regulated variables, $w_i \epsilon \mathcal{R}^{r_i}$ is exogenous signal for i^{th}-subsystem. Moreover, assume that there is no unstable fixed mode with respect to

$$[C^T{}_{y1}, C^T{}_{y2}, \cdots, C^T{}_{yN}], [A_{ij}]_{N\times N}, [B_{21}^T \quad B_{22}^T \cdots \quad B_{2N}^T]^T$$

Consider the following decentralized output feedback PSS controller for the system given in eq. (28) by using eq. (16).

$$\dot{x}_{ci(t)} = A_{ci}x_i(t) + B_{ci}y_i(t)$$

$$u_i(t) = C_{ci}xc_i(t) + D_{ci}y_i(t) \quad (29)$$

Where $x_{ci} \epsilon \mathcal{R}^{n_{ci}}$ is the state of the i^{th}-local controller, $n_{ci}$is a specified dimension, and A_{ci}, B_{ci}, C_{ci}, D_{ci}, i =1, 2,.., N are constant matrices to be determined during the designing. In this paper, the design procedure deals with nonzero D_{ci}, however, it can be set to zero, i.e., D_{ci} = 0, so that the i^{th}-local is strictly proper controller. After augmenting the decentralized controller eq. (29) in the system, the state space equation for the i^{th}-subsystem will have the following form: [11]

$$\dot{\tilde{x}}_i(t) = (\tilde{A}_{ii} + \tilde{B}_{2i}K_i\tilde{C}_{yi})\tilde{x}_i(t) + (\tilde{B}_{ij} + \tilde{B}_{2i}K_i\tilde{C}_{yi})w_i(t) + \sum_{j\neq i}\tilde{A}_{ij}\tilde{x}_j(t) \quad (30)$$

$$z_i(t) = (\tilde{C}_1 + \tilde{D}_{12}K_i\tilde{C}_{yi})\tilde{x}(t) + (\tilde{D}_{11} + \tilde{B}_{12}K_i\tilde{C}_{y1})w_i(t)$$

Where $\tilde{x}_i(t) = [x_i^T(t) \quad x_{ci}^T(t)]^T$, is the augmented state variable for the i^{th}-subsystem and

$$\tilde{A}_{i1} = \begin{bmatrix} A_{ij} & 0_{n_i\times n_{ci}} \\ 0_{n_{ci}\times n_i} & 0_{n_{ci}\times n_{ci}} \end{bmatrix}, \tilde{B}_{1i} = \begin{bmatrix} B_{1i} \\ 0_{n_i\times r_i} \end{bmatrix},$$

$$\tilde{B}_{2i} = \begin{bmatrix} 0_{n_{ci}\times n_{ci}} & B_{2i} \\ I_{n_{ci}} & 0_{n_{ci}\times n_{mi}} \end{bmatrix},$$

$$\tilde{c}_{yi} = \begin{bmatrix} 0_{n_{ci}\times n_{ci}} & I_{n_{ci}\times n_{ci}} \\ C_{yi} & 0_{q_i\times n_{ci}} \end{bmatrix},$$

$$\tilde{C}_{1i} = [C_{1i} \quad 0_{p_i\times n_{ci}}],$$

$$K_i = \begin{bmatrix} A_{ci} & B_{ci} \\ C_{ci} & D_{ci} \end{bmatrix}, \quad \tilde{D}_{11i} = D_{11i}$$

$$\tilde{D}_{12i} = [0_{p_i\times n_{ci}} \quad D_{12i}], \quad \tilde{D}_{y1i} = \begin{bmatrix} 0_{n_{ci}\times r_i} \\ D_{y1i} \end{bmatrix}$$

Moreover, the overall extended system equation for the system can be rewritten in one state-space equation form as

$$\dot{\tilde{x}}_i(t) = (\tilde{A}_{ii} + \tilde{B}_{2i}K_i\tilde{C}_{yi})\tilde{x}_i(t) + (\tilde{B}_{ij} + \tilde{B}_{2i}K_i\tilde{C}_{yi})w_i(t)z_i(t) = (\tilde{C}_1 + \tilde{D}_{12}K_i\tilde{C}_{yi})\tilde{x}(t) + (\tilde{D}_{11} + \tilde{B}_{12}K_i\tilde{C}_{y1})w_i(t) \quad (31)$$

Where,

$$\tilde{A} = [\tilde{A}_{ij}]_{N\times N},$$
$$\tilde{D}_{11} = \text{diag}\{\tilde{D}_{111}, \tilde{D}_{112}, \cdots, \tilde{D}_{11N}\}$$
$$\tilde{D}_{12} = \text{diag}\{\tilde{D}_{121}, \tilde{D}_{122}, \cdots, \tilde{D}_{12N}\},$$
$$\tilde{D}_{y1} = \text{diag}\{\tilde{D}_{y11}, \tilde{D}_{y12}, \cdots, \tilde{D}_{y1N}\},$$
$$\tilde{B}_1 = \text{diag}\{\tilde{B}_{11}, \tilde{B}_{12}, \cdots, \tilde{B}_{1N}\},$$
$$\tilde{B}_2 = \text{diag}\{\tilde{B}_{21}, \tilde{B}_{22}, \cdots, \tilde{B}_{2N}\},$$
$$\tilde{C}_y = \text{diag}\{\tilde{C}_{y1}, \tilde{C}_{y2}, \cdots, \tilde{C}_{yN}\},$$
$$\tilde{C}_1 = \text{diag}\{\tilde{C}_{11}, \tilde{C}_{12}, \cdots, \tilde{C}_{1N}\},$$
$$K_D = \{K_1, K_2, \ldots., K_N\}.$$

Where,

$$A_{ci} = \tilde{A}_{ii} + \tilde{B}_{2i}K_i\tilde{C}_{yi},$$

$$B_{ci} = \tilde{B}_{ij} + \tilde{B}_{2i}K_i\tilde{C}_{y1},$$

$$C_{ci} = \tilde{C}_1 + \tilde{D}_{12}K_i\tilde{C}_{yi},$$

$$D_{ci} = \tilde{D}_{ij} + \tilde{B}_{2i}K_i\tilde{C}_{y1}$$

Consider the following design approach where the controller strategy in eq. (22) internally stabilizes the closed-loop of the transfer function $\|T_{zw}(s)\|$ from w to z and moreover satisfies a certain prescribed disturbance attenuation level $\gamma > 0$, i.e., $\|T_{zw}(s)\|_\infty < \gamma$. In the following, the design procedure assumes that the system in eq. (21) is stabilizable with the same prescribed disturbance attenuation level γ via a centralized H_∞ controller of dimension equal to or greater than $n_c = \sum_{i=1}^{N} n_{ci}$ in which each controller input $\mathbf{u}_i$is determined by all measured outputs y_j, $1 \leq j \leq N$. The significance of this assumption lies on the fact that the decentralized controllers cannot achieve better performances than that of centralized controllers. In this paper, the centralized $H\infty$ controller is used as initial value in the two-stage matrix inequality optimization method.

4.2 Decentralized H_∞ Output Feedback Design

Designing a decentralized H_∞ output feedback controller for the system is equivalent to that of finding the matrix $\widetilde{K}_D$ that satisfies an H_∞ norm bound condition on the closed-loop transfer function $T_{zw0}(s)$ = $C_{cl0}(sI - A_{cl0})B_{cl0}+D_{cl0}$from disturbance $w_0$to measured output z, i.e. $\|T_{zw}(s)\|_\infty < \gamma$ (for a given scalar constant $\gamma > 0$). Moreover, the transfer functions $T_{zw}(s)$ must be stable [12]. The instrumental in establishing the existence of decentralized control strategy eq. (22) that satisfies a certain prescribed disturbance attenuation level $\gamma > 0$ on the closed loop transfer function $T_{zw}(s)$ = $\mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl}+\mathbf{D}_{cl}$ from disturbance w to measured output z, i.e. $\|Tzw\|_\infty < \gamma$.

However, due to the specified structure on the controller (i.e., designing controllers with "block diagonal") makes the problem a nonconvex optimization problem. To compute the optimal solution of this problem, the design problem is reformulated as an embedded parameter continuation problem that deforms from the centralized controller to the decentralized one as the continuation parameter monotonically varies [13]. The parameterized family of the problem is given as follows:

$$\widetilde{\Phi}(K_D, \widetilde{P}, 0) = \Phi\left((I-\lambda)K_F + \lambda K_D, \widetilde{P}\right) < 0 \qquad (32)$$

with $\lambda \epsilon [0,1]$ such that at $\lambda = 0$

$$\widetilde{\Phi}(K_D, \widetilde{P}, 0) = \Phi(K_F, \widetilde{P}) \text{ at } \lambda = 1 \qquad (33)$$

$$\widetilde{\Phi}(K_D, \widetilde{P}, 0) = \Phi(K_D, \widetilde{P}) \qquad (34)$$

$$\text{Where } K_F = \begin{bmatrix} A_F & B_F \\ C_F & D_F \end{bmatrix} \qquad (35)$$

is a constant matrix of the same size as K_D and composed of the coefficient matrices A_F, B_F, C_F and D_F of an n_c dimensional centralized H_∞ for the disturbance attenuation level γ . The centralized controller K_F can be obtained via the existing method. Thus, the term $(1-\lambda)K_F+\lambda K_D$ in eq. (32) defines a homotopy interpolating centralized H_∞ controller and a desired decentralized H_∞ controller. Thus, the problem of finding a solution of eq. (26) can be embedded in the family of problems as:

$$\widetilde{\Phi}(K_D, \widetilde{P}, \lambda) < 0\,, \quad \lambda \epsilon [0,1] \qquad (36)$$

Thus, the algorithm based on parameter continuation method for finding the robust decentralized output feedback controller has the following two-stage matrix inequality optimization algorithm [11].

4.3 Reduced Order Decentralized Controller Design

The algorithm proposed in the previous section can only be applied when the dimension of the decentralized H_∞ controller is equal to the order of the plant, i.e., $n=n_c$. However, it is possible to compute directly a reduced-order decentralized controller, i.e., $n_c < n$ by augmenting the matrix $K_D$as

$$\grave{K}_D = \begin{bmatrix} \grave{A}_D & 0_{n_n \times (n-n_c)} & \vdots\ \acute{B}_D \\ * & -I_{n-n_c} & \vdots\ ** \\ \cdots & \cdots & \cdots\ \cdots \\ \grave{C}_D & 0_{m \times n-n_c} & \vdots\ \acute{D}_D \end{bmatrix} \qquad (37)$$

Where the notation $*,**$ are any submatrices $\grave{A}_D$, $\acute{B}_D$, $\grave{C}_D$, and $\acute{D}_D$ are the reduced-order decentralized controller matrices. Note that the n-dimensional controller defined by $\grave{K}_D$ of eq. (40) is equivalent to the n_c-dimensional decentralized controller described by state-space representation of ($\grave{A}_D$, $\acute{B}_D$, $\grave{C}_D$, $\acute{D}_D$) if the controller and observable parts are extracted. Next, define the matrix function $\widetilde{\Phi}(K_D, \widetilde{P}, \lambda)$as

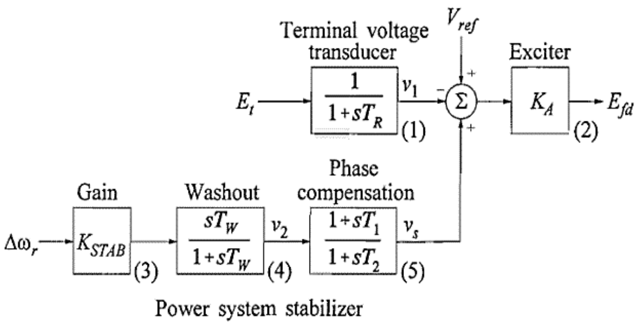
$$\widetilde{\Phi}(\widetilde{K}_D, \widetilde{P}, \lambda) = \Phi\left((I-\lambda)K_F + \lambda \widetilde{K}_D, \widetilde{P}\right) < 0 \qquad (38)$$

Then, one can apply the algorithm proposed in the previous section with $K_F$of n-dimensional centralized H_∞ controller. In this case, at λ =0 set the matrix $\widetilde{K}_D$ to zero except (2,2)- block-I $_{(n-nc)}$and proceed with computing $\grave{K}_D$ for each λ. If the algorithm succeeds, then the matrices ($\grave{A}_D$, $\acute{B}_D$, $\grave{C}_D$ and $\acute{D}_D$) extracted from the obtained $\grave{K}_D$at λ =1 , comprise the desired decentralized H_∞ controller [9].

5. SIMULATION RESULTS

The robust decentralized dynamic output control design approaches presented in the previous section of this paper are now applied to a test system. This system is Sudan grid, which consists of 41 generators and 167 bus bars specifically designed to study the fundamental behavior of large interconnected power systems including inter-area oscillations. The system has 41 generators and each generator is equipped with standard exciter and governor controllers. The parameters for the standard exciter and governor controllers used in the simulation were taken from Sudan grid companies. Moreover, the generators for these simulations are all represented by their fifth-order models with rated terminal voltage of 10.5 KV, 11 kV and 13.8 kV. In the design, speed signals from each generator are used for robust decentralized dynamic output control through the excitation systems. The following loading condition was assumed: at bus 28-63 transmission line (Khartoum-Kuku) a load of [PL1 104 MW, QL1 56 Mvar]. The system was linearized and the corresponding system equations were decomposed as a sum of two sets of equations. While the former describes the system as a hierarchical interconnection of 41subsystems, the latter represents the interactions among the subsystems. After augmenting the controller structure in each subsystem, the design problem was formulated as a minimization problem of linear objective function involving LMIs and coupling BMI constraints. Using the algorithm proposed in, the robust sub-optimal decentralized second-order PSS for each subsystem were designed. The robust decentralized PSS designed through this approach are given in Table-1. The convergence behavior of Algorithm for a relative accuracy of ε = 10−6. Similarly, second-order PSS are designed using the approach proposed. However, it is possible to extend the method to any order and/or combinations of PSS blocks in the design procedure. After including the washout filter in the linearized system equation, the design problem is reformulated as an embedded parameter continuation problem that deforms.

From the designed centralized H_∞ controller to the decentralized one as the continuation parameter monotonically varies. Speed signals from each generator, the outputs of the PSS together with the terminal voltage error signals, which are the input to the regulator of the exciter, are used as regulated signals within this design framework. The robust decentralized PSS designed through this approach are also given in table-1. For loss line which connects bus bars 28-63 at (places), the transient responses of generators on Marrwi electrical-station with and without the PSSs in the system are shown in Figs.(3) to Figs (7) To further assess the effectiveness of the proposed approaches regarding the robustness, the transient performance indices were computed for keeping constant total load in the system. The transient performance indices for generator angle θ_{gi} Fig (3), generator speed deviation ω_{gi} Fig (4) generator powers Pg_i Fig (5), generator terminal voltages V_{ti} Fig (6), and Line power flow from generator Fig (7) following when loss line at bas bars 28-63 at (places), computed using the Matlab Power System Toolbox Author: Graham Rogers, which call load flow programs without PSS and with PSS acted as feedback controller. These transient performance indices are used as a qualitative measure of system behavior following any disturbances including controller actions. Moreover, for comparison purpose, these indices are normalized to the base operating condition for which the controllers have been designed: Any power system stabilizer has one input power



Assumed values for the parameters

$T_W = 1.40\ S$ $\qquad T_{i2} = 0.29\ S$

Fig. 2. The PSS structure used in the design

Table 1. The calculated PSS gains and parameters for each generator

Generator No	Gains for the PSS K_{PSS}	Washout time constant$T_{f.lead}$	First lead time $T_{f.lead}$	First lag time $T_{f.lead}$	Second lead time $T_{f.lead}$	Second lag time $T_{f.lead}$
1-3	100	10	0.1	0.02	0.08	0.02
4-7	50	10	0.1	0.02	0.08	0.02
8-20	100	10	0.1	0.02	0.08	0.02
21-27	10	10	0.1	0.02	0.08	0.02
28-29	50	10	0.1	0.02	0.08	0.02
30	10	10	0.1	0.02	0.08	0.02
31	10	10	0.1	0.02	0.08	0.02
32-41	100	10	0.1	0.02	0.08	0.02

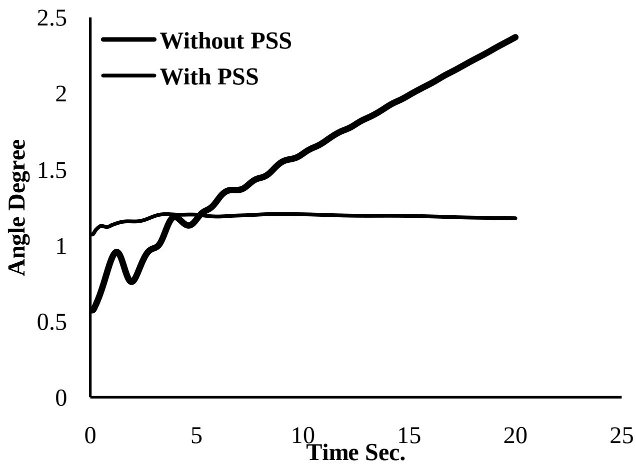


Fig. 3. Machine angle of Marrwi hydro-turbine without and with PSS

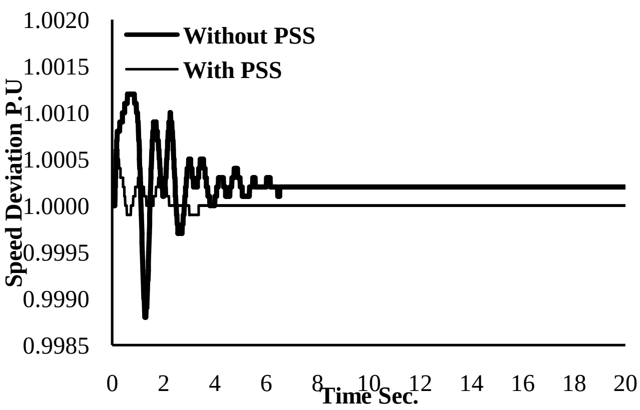


Fig.4. Machine speed deviation of Marrwi hydro-turbine without and with PSS

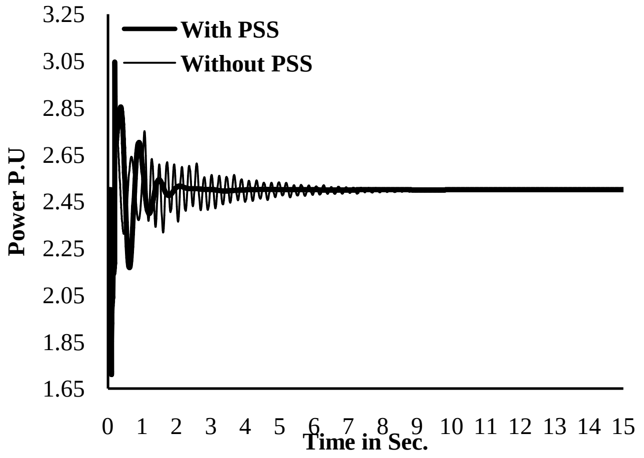


Fig. 5. Machines electrical power of Marrwi hydro-turbine without and with PSS

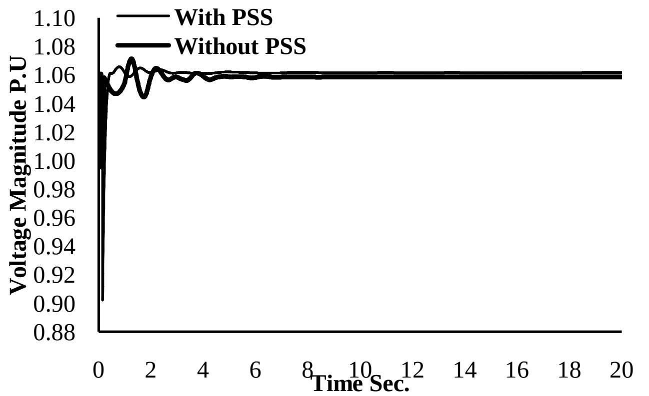


Fig. 6. Machines voltage magnitude of Marrwi hydro-turbine without and with PSS

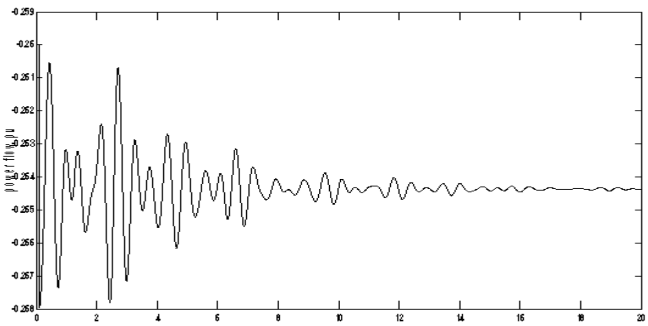


Fig. 7. Line power flow of Marrwi hydro-turbine without PSS

6. CONCLUSIONS

The objective of power system stabilizers is to extend stability limits on power, transfer by enhancing damping of system oscillations via generator excitation control. Lightly damped oscillations can limit power transfer under weak system conditions, associated with either remote generation transmitting power over long distances or relatively weak interties connecting large areas. This control must include not only the transient damping contributions to all modes of system oscillation, but the impact upon system performance following large disturbances and line outer without fault, when all nodes of the system are excited simultaneously. Stabilizers utilizing inputs of speed, power, and frequency have been analyzed with respect to both tuning concepts and performance capabilities.

A robust damping controller for power system oscillations is presented. The robust controller is a supervisory level controller that can track system inter-area dynamics online. An LMI-based method is applied to design H_∞ controllers. The PSS controller structure may significantly increase the system operating range. The performance of the Sudan grid robust controller is illustrated using a 41 machine, 167-bus system. Based on limited testing, the simulation results show that the proposed robust controller can effectively damp system oscillations under range line outer without fault conditions.

In this paper, a design scheme of power system stabilizers for a multi-machine (41 generators) power system with 167 buses using decentralized fast output sampling feedback via reduced order model has been developed. The proposed method results satisfactory response behaviour to damp out the oscillations. The decentralized control via reduced order model can be applied simultaneously to the all machines. Thus the applied control scheme is of decentralized nature. This method can be extended to design the robust decentralized power system stabilizers for a multi-machine power system. It is found that designed controller provides good damping enhancement for various operating points of Sudan grid. The proposed controller results in a better response behavior to damp out the oscillations. Simulation results from a nonlinear power system are given to demonstrate the applicability and effectiveness of the proposed approach. This paper presents a linear matrix inequality (LMI)-based approach for designing a robust decentralized structure-constrained controller for power systems. The problem of designing a fixed-structure H_2/H_∞dynamic output feedback controller is first reformulated as an extension of a static output feedback controller design problem for the extended system. The resulting optimization problem has bilinear matrix inequalities (BMIs) form which is solved using sequentially LMI programming method. The effectiveness of the proposed approach is demonstrated by designing (sub)-optimal fixed-structure power system stabilizers (PSSs) controllers for a test power system so as to determine the optimal parameters.

The paper also proposes a new approach to solve such optimization problems using a sequential LMI programming method to determine (sub)-optimally the decentralized output dynamic feedback controllers of the system. The approach is demonstrated by designing power system stabilizers (PSSs) for a test power system.

The testing of this system showed positive results. The LMI controller worked well and provided a reasonable amount of damping to the inter-area modes.

REFERENCES

[1] P. Kundur, Power System Stability and Control. McGraw-Hill, Inc. Newyork, 1994.

[2] P. M. Anderson and A. A. Fouad, Power System Control and Stability. JohnWily and Sons, Inc., Publication United States,2003.

[3] K. R. Padiyar. Power System Dynamics Stability and Control. Second Edition,B S Publications, Hyderabad, 2002.

[4] A. S. R. Ramamurthy, S. Parameswaran, and K. Ramar. Design of decentralized variable structure stabilizers for multimachine power systems. Electrical Power and Energy Systems, 1996.

[5] P. Shrikant Rao and I. Sen. Robust pole placement stabilizer design using linear matrix inequalities. IEEE Transactions on Power Systems, February 2000.

[6] G. K. Befekadu and I. Erlich, Robust Decentralized Structure – Constrained Controller Design for Power Systems: An LMI Approach, 15th PSCC, Liege, 22-26 2005

[7] George E. Boukarim, Shaopeng Wang, Joe H. Chow, Fellow, IEEE, Glauco N. Taranto, Member, IEEE, and Nelson Martins, Fellow, IEEE, A Comparison of Classical, Robust, and Decentralized Control Designs for Multiple Power System Stabilizers, IEEE Transactions on Power Systems, Vol.15,No.2000,

[8] Hui Ni, Gerald Thomas Heydt, Fellow, IEEE, and Lamine Mili, Senior Member, IEEE, Power System Stability Agents Using Robust Wide Area Control, , IEEE Transactions on Power Systems, Vol. 17, No.4 Novermber 2002,

[9] Da-Wei Gu _ Petko H. Petkov _Mihail M. Konstantinov Robust Control Design with MATLAB Second Edition © Springer-Verlag London 2005, 2013

[10] Wenzheng Qiu, Vijay Vittal, and Mustafa Khammash, Senior, Decentralized Power System Stabilizer Design Using Linear Parameter Varying Approach, IEEE Transactions on Power Systems, Vol. 19, No. 4, Novermber 2004,

[11] K Mocwane, KA Folly, Robustness Evaluation Of H_2/H_∞ Power System Stabilizer, IEEE PES Power Africa 2007 - Conference and Exhibition Johannesburg, South Africa, 2007

[12] M. Bouhamida , A. Mokhtari, M. A. Denai, Power System Stabilizer Design Based on Robust Control Techniques, ACSE Journal, Volume (5), Issue (3), Nov., 2005

[13] Fadhil M. Kalaf, The Effect of Power System Stabilizers (PSS) on Synchronous Generator Damping and Synchronizing Torques, Al-Mansour Journal Issue (18) 2012.