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A Mathematical Model to Predict Hydraulic Fracture Geometry in Shale Gas Formation

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Abstract: Shale gas represents a new abundance for natural gas, which could aid keeping up with increasing world demand for energy. Enormous quantities of shale gas reserves are in Africa and China; however the technology to extract natural gas from shale formations is not yet developed in these regions. Hydraulic fracturing is the most viable technique to recover shale gas in economical quantities. Designing and controlling fracture geometry are crucial to achieve economical production rates from shale gas formations. Currently there is no universal model that can be applied to predict the fracture height and half-length in shale gas formation. Earlier models have addressed the issue of fracture geometry that is induced by hydraulic fracturing operation in conventional formations. Models such as KGD (Khristianovich- Geertsma-DeKlerk) and PKN (Perkins-Kern-Nordgren) solving the problem by assuming fixed fracture height, while complicated 3D models (planar and pseudo) are used to describe fracture geometry. Pseudo-3D model suffers from unrealistic fracture height outputs when the assumptions are violated. In this paper new equations for both fracture's height and half-length are developed using Bingham theory in combination with a statistical approach (Monte Carlo simulation). The equations can be applied for wide ranges of rock properties and operational conditions of hydraulic fracturing.Model's outcomes validation was verified using available data in the literature. Moreover, a parametric study showed that fluids viscosity and Poisson's ratio were the major parameters that control fracture half-length. On the other hand, fracture height has been found to predominantly be controlled by formation thickness. These results are in line with the previous models outcomes.

Keywords: *Hydraulic fracturing; Shale gas; fracture half length; fracture height, Monte Carlo Simulation Econometric model.*

1. INTRODUCTION

Global demands of natural gas are increasing, IEA anticipated that global energy demands will raise by 37% by 2035[1]. This consequently raises the need to discover new resources of energy. Shale gas represents a new abundance of natural gas supply. Global technically recoverable resources from shale gas are estimated to be 6,622 trillion cubic feet [2]. Shale gas is the natural gas that resides in a fine-grained sedimentary rock, known as shale. It is found in deep shale formation layers with very low porosity and permeability because of high rock compressibility resulting from overburden pressure. Shale gas cannot be recovered in economic quantities unless the technique of hydraulic fracturing is used in combination with horizontal drilling in most cases. Currently, hydraulic fracturing technique is the only means to produce Natural Gas from Shale formations in commercial volumes, by simulating gas wells through creating fractures within the shale matrix in order to secure paths for Natural Gas to flow from the formation layer into the wellbore.

1.1. Geometry of Hydraulic Fractures

Fractures are initiated by injecting fluids (usually slick water) into the intended formation at a rate higher than the fluids can leak off into the formation. The high injection rate consequently builds up a fluid pressure sufficient to overcome the formation rock compressive pressure to create fractures network. The fractures are created perpendicular to the minimum horizontal stress. Hence to fracture a formation, the injection pressure must exceed the minimum stress. This fracture grows in three dimensions (width, height, and half-length) which make up fracture geometry. Understanding and modeling the fracture geometry is crucial for a successful hydraulic fracturing procedure. Most models that tackled this issue in conventional formations are based predominantly on KGD and PKN models which assume that the fracture is planar (fracture propagates in a particular direction). Perkins and Kern developed equations to compute fracture length and width with a fixed height [3]. Later Nordgren improved this model by adding fluid loss to the solution [4]. This model is commonly called PKN model [5]. On the other hand, KGD model was developed by Khristianovitch and Zheltov and Geertsma and de Klerk. In this model, plane strain is assumed to be in horizontal direction i.e., all horizontal cross sections act independently so that fracture height, h_f, must be much greater than fracture length [5] [6]. KGD also assumes that the fracture height is constant and the effect of fracture toughness is negligible, which is defined as the rock resistance for fracture propagation. Additionally, both models don't include the effect of fracture net pressure (pressure inside the fracture minus the in-situ stress) in their calculations for fracture half-length, x_f.

Many models have addressed the issue of fracture geometry design for shale formations to idealize the hydraulic fracture performance. In order to increase the accuracy of fracture geometry estimations, 3D models have been developed [7]. One of these models is the Pseudo-3D model which assumes that the fracture height is always greater than or equal to the formation thickness along the whole fracture length. The equilibrium height equation suffers from the condition of "run-away" height when the assumptions made are not valid [5] [8].The other 3D model is the planar-3D which assumes that the fracture is planar and perpendicular to the minimum principal stress. This model uses complex numerical equations to predict fracture geometry which require complex simulators due to the nonlinear relation between fracture width and pressure. Therefore, this model is costly, time consuming, cannot be used in a routine fracture design, and should only be used when a portion of the fracture shows significant homogeneity [9].

Eventually it is concluded that there are numerous issues regarding fracture geometry that need to be addressed especially when it comes to design hydraulic fracturing job in unconventional shale gas formations. It is important to consider the issue of fracture height "Run Away" condition in the Pseudo-3D model, as well as the absence of an explicit equation describing fracture half-length. Although planar 3D model covers most of these concerns, yet it is still a very complicated, time consuming, and costly model [10].

In this paper, the target is to obtain a universal fracture geometry model thatprovidessimple, accurate equations for fracture height and half-lengthwhich can be applicable for fracking shale formation with horizontal well.

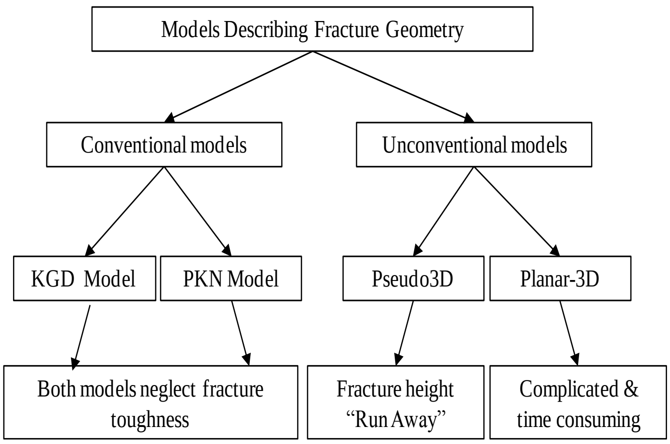

Fig. 1. Summary of the most common models used to simulate fracture geometry

2. METERIALS AD METHODS

2.1. Model Formulation

In order to develop a mathematical model describing fracture geometry in shale formation the following steps were followed:

First, an intensive study was conducted to comprehend the major parameters that control fracture height and half-length. These parameters are summarized in **Table1**. [11] [12] [13].

In the following step Bingham theory has been utilized to derive two equations for fracture height and fracture half length, respectively. These equations are in dimensionless form as follow:

$$\frac{h_f}{h} = R * \left(\frac{P_{net}}{E}\right)^A * \left(\frac{K_{IC}}{E\sqrt{h}}\right)^B \quad (1)$$

$$\frac{x_f}{h_f} = R'\left(\frac{q_i}{h_f^3}\right)^{A'} * \left(\frac{E'}{\mu}\right)^{B'} * \left(\frac{p_{net}}{\mu}\right)^{C'} * \left(\frac{K_{IC}}{\mu\sqrt{h_f}}\right)^{D'} * t_i^{E'} \quad (2)$$

In order to determine the coefficients in Equations 1 and 2, sufficient data is required. Since there is little available data in the literature about these parameters tested and measured in fracking shale formations; therefore all available data ranges of each parameter was modeled using Monte Carlo Simulation technique.

2.2. Statistical Method (Monte Carlo Simulation)

Monte Carlo simulation is a numerical modeling technique, named after the city of Monte Carlo in Monaco, where the primary attractions are casinos that play games of chance like dice, cards and others. It is a technique that uses a random number generator to produce and extract an uncertain variable within a distribution model for calculation in a given formula or correlation.

In this approach and in order to fill the data gap, all the possible values for both rock properties and hydraulic fracturing operating parameters in shale formation were used as shown in **Table 2**. The next step is to perform number of iterations to understand the range in which the coefficients may be apply for shale formations.Monte Carlo Simulationwas utilized to obtain the coefficients which have the highest probability to give the required results.

Table 1. Major parameters that are controlling fracture's height and half length

Parameters that affect fracture height (h_f)	**Parameters that affect fracture half-length (x_f) in addition to fracture height**
Young's modulus (E)	Fracture (treatment) net pressure(*psi*)
Fracture net pressure (P_{net}) measured in (*psi*)	Plain strain modulus, (*psi*)
Fracture toughness (K_{IC}) measured in ($psi.\ in^{0.5}$)	Fracture toughness, ($psi.in^{0.5}$)
Injection rate (q_i) measured in (*bpm*)	Injection rate, (*bpm*)
Formation thickness (h) measured in (*ft*)	Injection fluid viscosity, (*cp*)

Table 2. Variation of rock properties in shale formations collected from the literature*

Parameter	Minimum Value	Maximum Value
Pore Pressure (psi)	3000	6500
Tectonic Stress(psi)	300	1200
Formation Depth (ft)	6200	9600
Formation Thickness(ft)	100	300
Poisson's Ratio	0.25	0.4
Young's Modulus(psi)	5000000	5800000
Fracture Toughness(psi-in$^{0.5}$)	1000	2000
Bulk Density (gm/cm3)	2.58	2.8
Injection Fluid Viscosity (cp)	1	10
Fluid Injection Rate (BPM)	20	100

* [14][15][16]

After deriving the coefficients and coming up with the final equations for fracture height and half-length, the fracture width equation has been adopted from the PKN model. This because it widely used by the petroleum industry in Shale fracturing design. The new universal model is shown as follow:

$$h_f = 1.86\left(\frac{P_{net}}{E}\right)^{00.26}\left(\frac{K_{IC}}{E\sqrt{h}}\right)^{-0.12} * h \quad (3)$$

$$x_f = 0.12h_f * \left(\frac{q_i}{h_f^3}\right)^{0.56}\left(\frac{E'}{\mu}\right)^{0.1666}\left(\frac{P_{net}}{\mu}\right)^{0.87}\left(\frac{K_{IC}}{\mu\sqrt{h_f}}\right)^{-0.3} t^{0.666} \quad (4)$$

$$w_{Max}. = \frac{2h_f(1-\upsilon^2)}{E} \quad (5)$$

$$w_{avg} = \frac{\pi}{4} w_{max}. \quad (6)$$

2.3. Model Results and Verification:

Following the derivation of Equations 3 and 4, they were tested in a wide range of shale formations with different properties and conditions. Subsequently, the variation of fracture geometries which have been observed from the study were recorded and interpreted by the aid of Monte Carlo Simulation.

It was observed that the new model can estimate wide ranges of fracture height and half-length with a good confidence. Furthermore, the model is applicable for shale formations despite the fact that the variation in rock properties can be used to predict fracture geometry.

Moreover, two case studies are presented below to verify the outcomes of applying this new model using available data from the literature. The two cases, which are from two different fields in the United State, have got satisfactory data to be applied in the new model and then the model's results will be compared with the actual available fracture geometry, i.e. the fracture half-length and height obtained from hydraulic fracturing process in these fields.

2.4. Case Study

Data were collected from Chesapeake Energy [17].**Tables 3** and **4**, depict available data and the assumed ranges for the missed data, respectively.Since the given data was missing some essential data for the model. Therefore the missing data is given in the format of all possible ranges in order to accommodate all uncertainties in these data.

3. RESULTSAND DISCUSSION

Again Monte-Carlo Simulation was used to apply the new model since the missing data are assumed in ranges with uniform probability distribution. Values of fracture's height and half-length are calculated using Equations 3 and 4 in terms of cumulative probability distribution. The results obtained from the model are compared with the actual values of *xf*and *hf*as per **Table 3**.Visual representation of results is given in **Figs 2**, in which P10, P50 and P90 are illustrated for the calculated*xf*and *hf*.

From the above case study one can conclude that by using the developed Equations 3 and 4, good and relatively accurate prediction to the fracture geometry can be obtained.Other cases found in the literature were studied and the same conclusion of quite accurate prediction of fracture geometry using this new model was reached.

Table 3. Data input for case study

Parameter	Value
Pore Pressure (psi)	3705.45
Formation Depth (ft)	6175.73
Poisson's Ratio	0.285
Young's Modulus (psi)	4391000
Plain Strain Modulus (psi)	4779189.68
Biot's Coefficient	0.72
Fracture Toughness(psi-in^0.5)	1500
Fracture Half-length (ft)	800
Fracture Height (ft)	250

Table 4. Assumed input data

Parameter	Minimum value	Maximum value
Tectonic stress (psi)	300	900
Formation thickness (ft)	200	300
Bulk density (gm/cm^3)	2.58	2.8
Injection fluid viscosity (cp)	1	10
Fluid injection Rate (BPM)	20	100

Table 5: Results for case study

Output Values	Fracture half-length (ft)	Fracture height (ft)
Actual value	800	250
Calculated value	798	216.5
Deviation from actual values	0.25%	13.4%

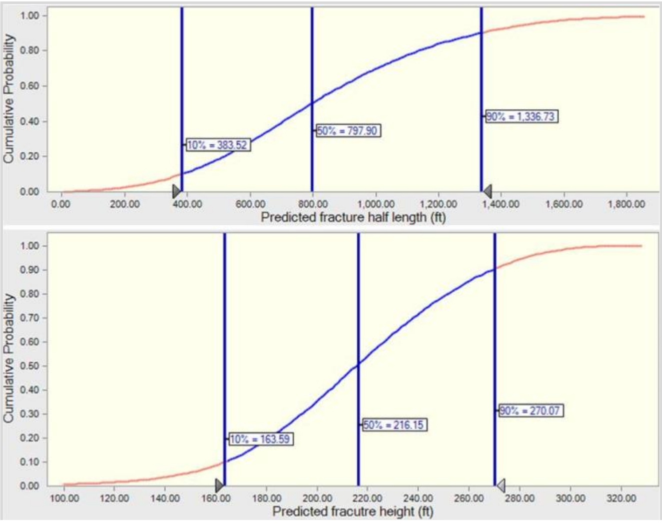

Fig. 2. Cumulative probability distribution for the calculated fracturehalf-length and height

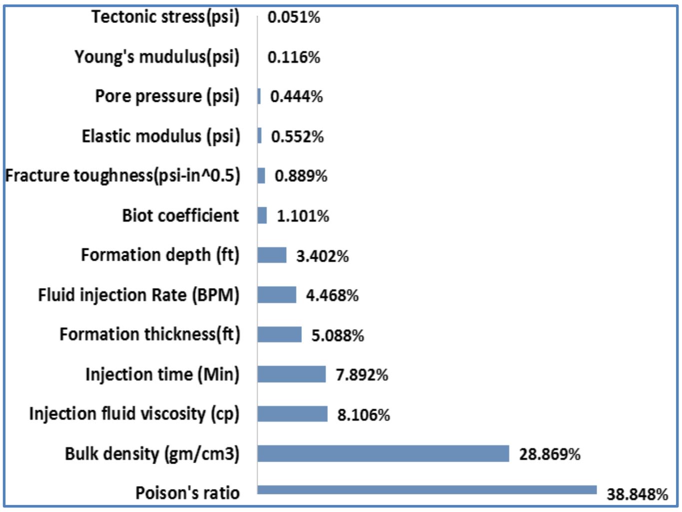

Fig. 3. Contribution of variance for different parameters on fracture half-length as calculated with Equation 4

This section discusses the results which have been obtained from analysis on the new equations, which particularly focus parametric study. The purpose of this study is to find out the effects of some of the rock properties and operational conditions on the expected model results. Finally, some limitations of the new model were addressed.

Again, Monte Carlo Simulation is used to apply sensitivity tests on both Equations 3 and 4. Different value ranges of the variables were assumed and contribution of variance was recorded to both $x_f$and h_f. It is also found that the uncertainty in bulk density and Poisson's ratio might remarkably affect the calculated half length, this is also in-line with previous studies regarding this issue. Also the effect of fracture toughness and Young's modulus was investigated were been found to have minor negative effects on fracture half length, which is very much in line with the previous studies concerning the effect of fracture toughness on the geometry. Parametric study also covered the rest of parameters and they are listed by their order of contribution to variance as in **Fig. 3.**

As for fracture height equation, it is observed that the dominant factor which controls the fracture height growth is the formation thickness. The Parametric study showed that formation thickness contribution to fracture height growth is almost 61%. However, analysis of height growth shows that fracture height can hardly overpass the thickness of the shale zone, except in extreme conditions. Moreover, it can be noticed that Poisson's ratio and bulk density might affect fracture height to some extent. Similar to previous studies, the effects of fracture toughness and Young's modulus are negligible. Other parameters which affect fracture height are presented in **Fig. 4**.

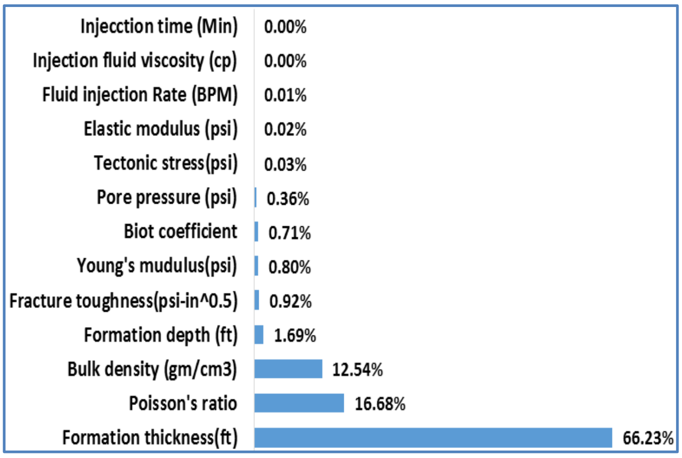

Fig. 4. Contribution of variance for different parameters on fracture height as calculated with Equation 3

4. CONCLUSIONS

- Fracture geometry models used for conventional reservoirs may not simulate fracture geometry in shale formation. A new model need to be developed and can be used to predict fracture half-length and height in shale gas formation.
- Dimensionless analysis is used together with the aid of Monte Carlo Simulation to develop new equations for both fracture height and half length. The developed model can estimate fracture height and half-length in wide range of rock properties variation in shale formations.
- The model is verified and proved to be useful to predict the fracture height and half length
- Sensitivity analysis on the different parameters of the developed fracture half-length equation indicates that uncertainty in bulk density and Poisson's ratio are found to be predominant factor that control the effectiveness of the fracture half length.
- The parametric study shows that fracture toughness and Young's modulus can be neglected due to their relatively low effect on fracture geometry.

- Fracture thickness is the major factor that controls fracture height growth, however it is unlikely that fracture height will bridge shale thickness.

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