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Open Access A Bohr-like model of the planetary system and its gravitoelectric characteristics

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Abstract
We have presented a quantum model of planetary system analogous to Bohr model of hydrogen atom. The quantization of the angular momentum of planets agrees with Newtonian calculations. We also derive the energy density and power delivered by the system employing the electro-gravity analogy.

Keywords: *Generalized Newton's law; Gravitomagnetism; Quantum model*

1 Introduction

In an earlier work we have shown that analogous formulae govern gravitation are similar to electromagnetism one. The deflection of light is shown to be analogous to the deflection of α-particles by the nucleus (Arbab 2010a). According to this analogy similar phenomena occurring in both disciplines and governed by similar formulae. Gravitomagnetic diploe precesses in the gravitomagnetic field in the same was as a magnetic dipole moment precesses in the presence of a magnetic field. Hence, Larmor frequency describes both phenomena. The gravitomagnetic diploe precession is manifested in the advance of perihelion of planets and periastron of binary pulsars (Arbab 2010b). While Lorentz force of a charged body encompasses both electric and magnetic forces, gravitomagnetic force is absent in Newton's law of gravitation. A Lorentz-like gravity law is then developed that symmetrizes Newton's law of gravitation. Having this symmetric nature of gravity and electricity, we have shown that some of the gravitational phenomena that couldn't be explained by Newton's law is now explainable by the Lorentz-like law of gravity (Arbab 2010c). We call the latter law the "*generalized Newton's law of gravitation*".

In this letter, we would like to apply our analogy, between electricity and gravity, to explore other phenomena occurring in electromagnetism that have counter analogues in gravity, and explain them in a similar fashion. Of these quantities are the interaction energies resulting from various gravitational interactions. Gravitoelectric and magnetic pressures of the gravitational field will be induced by the Sun on the planets. These represent a measure of the energy density of the gravitoelectric and magnetic fields. Gravitational waves will not be developed since these fields are static.

2 Bohr-like model of planetary system

Consider a planet of mass m orbiting the Sun of mass M at a distance r with velocity v. The centripetal acceleration of the planet is given by

$$\frac{mv^2}{r}=\frac{GmM}{r^2}.$$

(1)

The total energy of the planet is the sum of the kinetic (K) and potential (U) energies, i.e.,

$$E=K+U=-\frac{GmM}{2r},$$

(2)

where

$$U=-\frac{GmM}{r},\qquad K=\frac{1}{2}mv^2.$$

(3)

In an analogous manner suppose now the angular momentum of the planet is quantized and is given by

$$L=mvr=n\hbar_c,$$

(4)

where $\hbar_c$ is a characteristic planck constant (Arbab 2004; 2005). Equations (1) and (2) yield

$$v_n=\left(\frac{GmM}{\hbar_c}\right)\frac{1}{n},$$

(5)

and

$$r_n=\left(\frac{\hbar_c^2}{GMm^2}\right)n^2.$$

(6)

Applying Eq.(6) in Eq.(2) yields

$$E_n=-\left(\frac{G^2M^2m^3}{2\hbar_c^2}\right)\frac{1}{n^2}.$$

(7)

We have earlier shown that the Planck constant for planetary system can be defined by $\hbar_c = \frac{GM^2}{c} = 8.8\times10^{41}\,Js$ (Arbab 2004; 2005)). Let us now write $\hbar_c = \beta\times10^{41}\,\mathrm{Js}$, where β is some constant. Equation (7) can be obtained from Bohr relations if we replace Ze by M, e by m and k by G, where

$$v_n = \left(\frac{k(Ze)e}{\hbar}\right)\frac{1}{n}, \qquad (8)$$

and

$$r_n = \left(\frac{\hbar^2}{k(Ze)^2e^2}\right)n^2. \qquad (9)$$

$$E_n = -\left(\frac{mk^2(Ze)^2e^2}{2\hbar^2}\right)\frac{1}{n^2} \qquad (10)$$

We can deduce the values of the quantum number n for every planet from Table 1 if β is somehow known.

Table 1: Angular momentum and total energy of planetary system.

Planet	βn	L_n **(J s)**	**Energy,** E_n **(** J **)**
Mercury	0.009152	9.15245 E+38	-3.79 E+32
Venus	0.184492	1.84492 E+40	-2.99 E+33
Earth	0.26609	2.6609 E+40	-2.65 E+33
Mars	0.035205	3.52048 E+39	-1.86 E+32
Jupiter	193.02	1.93022 E+43	-1.62 E+35
Saturn	78.297	7.82973 E+42	-2.64 E+34
Uranus	16.927	1.69276 E+42	-2.01 E+33
Neptune	0.0250	2.50188 E+42	-1.51 E+33
Pluto	0.004038	3.80837 E+38	-1.53 E+29

3 Energy and Power

We have recently found a Zeeman - like effect in gravitation that results from the interaction of gravitational dipole moment with gravitomagnetic field. This interaction energy is given by (Arbab 2010c)

$$E_{int.} = -\vec{\mu}_g \cdot \vec{B}_g = -\frac{v^6 r}{2Gc^2} = -\frac{1}{2} Mc^2 \left(\frac{v}{c}\right)^4,$$

(11)

while the gravitomagnetic power delivered by the mass in its orbit is given by

$$P_{gm} = I_g V_g = \frac{1}{4\pi Gc^2} v^7,$$

(12)

where

$$B_g = \frac{v^3}{rc^2}, \qquad \mu_g = \frac{v^3 r^2}{2G} = \frac{M}{2m} L, \qquad L = mvr.$$

(13)

The relativistic energy E of a particle of rest mass m_0 can be expanded as

$$E = \left(m_0 c^2 + \frac{1}{2} m_0 c^2 \left(\frac{v}{c}\right)^2 + \frac{3}{8} m_0 c^2 \left(\frac{v}{c}\right)^4 \right).$$

(14)

Comparing Eq.(11) with Eq.(14) reveals that the Zeeman energy is a second order correction to the relativistic energy. This comparison suggests that the factor $\frac{1}{2}$ should be replaced by $\frac{3}{8}$. It is interesting to notice that this energy is independent of the mass of the orbiting body. In electromagnetism one calculates the intensity of the electromagnetic radiation by the formula

$$I_e = \frac{1}{2} \varepsilon_0 c E^2, \qquad I_m = \frac{B_e^2 c}{2\mu_0}.$$

(15)

According to our analogy, the gravitoelectric and gravitomagnetic intensities should read

$$I_{gr} = \frac{1}{2} \varepsilon_{0g} E_g^2 v, \qquad I_{gm} = \left(\frac{B_g^2}{2\mu_{0g}}\right) v.$$

(16)

Equation (13) and (16) yield the relation

$$I_{gm}=\frac{v^7}{2Gc^2}\frac{1}{4\pi r^2},\qquad \mu_{0g}=\frac{4\pi G}{c^2}.$$

(17)

But since

$$I_{gm}=\frac{P_{gr}}{A},\qquad A=4\pi r^2,$$

(18)

one can write the gravitomagnetic power as

$$P_{gm}=\frac{v^7}{2Gc^2}.$$

(19)

Thus, apart from the factor $\frac{1}{2\pi}$, Eqs.(12) and (19) are identical.

The gravitoelectric intensity that is analogous to electromagnetic radiation intensity is given by

$$I_{gr}=\frac{1}{2}\varepsilon_{0g}vE_g^2=\frac{v^5}{8\pi Gr^2},\qquad E_g=a=\frac{v^2}{r},$$

(20)

which implies that the gravitoelectric power is

$$P_{gr}=\frac{v^5}{2G}.$$

(21)

It is interesting to notice that this power is independent of the mass of the orbiting body. The power delivered by the Sun that orbits the Milky Way with speed 220km/s amounts to $3.86\times10^{36}W$. This concides with the Milky way luminosity. It is known that the maximal power that can be generated from any physical system is (Massa 1995; Arbab 2004)

$$P_{max.}=\frac{c^5}{2G}.$$

(22)

This is thus a limiting value of Eq.(21).

The gravitational energy density is given by

$$u_{gr}=\frac{1}{2}\varepsilon_{0g}E_g^2,\qquad E_g=a.$$

(23)

Equation (20) can be written as

$$u_{gr}=\frac{a^2}{8\pi G}=\frac{v^4}{8\pi G r^2},\qquad \varepsilon_{0g}=\frac{1}{4\pi G}.$$

(24)

Equation (22) can be written as

$$u_{gr}=\frac{GM^2}{8\pi}\frac{1}{r^4},\qquad v=\sqrt{\frac{GM}{r}}.$$

(25)

This can also be interpreted as the pressure exerted by gravitational field of the Sun on Earth. And since the Earth is moving in its orbit, a gravitomagnetic pressure arising from gravitomagnetic field created by the Sun at the Earth site will be defined by the gravitomagnetic energy density. The gravitomagnetic energy density is given by

$$u_{gm}=\frac{B_g^2}{2\mu_{0g}}.$$

(26)

Using Eqs.(13) and (23) this becomes

$$u_{gm}=\left(\frac{v}{c}\right)^2 u_{gr}.$$

(27)

The total energy density is their sum, viz., $u=u_{gr}+u_{gm}$.

The electromagnetic radiation intensity emitted by a black body is related to its absolute temperature T by

$$I_{em}=\sigma T^4,$$

(28)

where σ is the Stefan's constant. The gravitoelectric intensity will thus be

$$I_{gr}=\frac{v^9}{8\pi G^3 M^2}.$$

(29)

If this has a black body spectrum, then, $T \propto v^{9/4}$. We remark here that while the intensity of the electromagnetic radiation emitted by a source mass (M) is specified by its surface temperature, the gravitational intensity is determined by the velocity of the orbital mass (m).

4 The gravitomagnetic force on a moving object and the resulting energy

According to our gravito-electric analogy, the gravitomagnetic force on a moving body (current) at a distance r from a central mass is given by

$$F_g = B_g I_g \ell, \qquad (30)$$

where (Arbab 2010c)

$$I_g = \frac{v^3}{2\pi G}, \qquad (31)$$

and $\ell = 2\pi r$ is the circumference of the orbiting body. The motion of an orbiting body can be compared with the current loop in a magnetic field. Once again, the gravitational current is independent of the mass of the orbiting body. Applying Eqs.(13) and (29) in Eq.(28) yields

$$F_g = \frac{v^6}{c^2 G}, \qquad (32)$$

It is postulated that there exists a maximal force in nature given by $F_{max.} = \frac{c^4}{G}$ (Massa 1995). This represents the limiting vale of Eq.(32). For a circular orbit $v = \sqrt{\frac{GM}{r}}$ so that Eq.(29) becomes

$$F_g = \frac{G^2 M^3}{c^2} \frac{1}{r^3}. \qquad (33)$$

This force is a tidal -like force that is manifested between any two gravitating objects. This force suggests that when an object is placed in a gravitomagnetic field it experiences a tidal force distorting it. The tidal-like force between the Earth and Sun is $1.18\times 10^{20}\,\text{N}$. This coincides with tidal force calculated using gravitational theory. The force exerted on the Moon is $1.85\times 10^{11}\,\text{N}$. This force may produce internal heat inside the body. Owing to this tidal-like force, some energy is dissipated inside the body. This energy can be found by integrating Eq.(31), viz.,

$$E_t = -\frac{G^2M^3}{2c^2}\frac{1}{r^2},$$
(34)

which is independent of the mass of the orbiting body. This energy is the same as the Zeeman energy defined in Eq.(11). For the Earth-Sun system, one has $E_t = -8.71\times10^{30}$ J. This is very close to the tidal dissipation energy which amounts to $(2-3)\times10^{30}$ J, or the radiogenic heat of 8.0×10^{30} J. For the Earth - Moon system, one has $E_t = -3.56\times10^{19}$ J. This amounts to a power of $W_t = 15\times10^{12}$ W which can be compared with the present tidal power of the Earth-Moon system which is about 3×10^{12} W. The tidal energy of a planet per orbit (period) is the same as the gravitomagnetic power in Eq.(19). We, thus, assume that when a gravitational dipole moment placed in a gravitomagnetic field the moment (planet) will orient itself in the field whereby it redistributes its internal mass that leads to the generation of heat inside the planet. Geologists assumed that the heat inside the planets (eg., Earth) is a result of the radiogenic energy of its composition. Table 2 shows the amounts of heat developed by planets in our solar system.

Table 2: Spin and Zeeman energy (heat) of planetary system.

Planet	mass ($\times10^{24}$ kg)	S (J s)	$E_{int.}$ (J)	Power (W)
Mercury	0.33022	1.79 E+31	-5.82 E+31	1.89 E+33
Venus	4.869	3.90 E+33	-1.66 E+31	3.95 E+32
Earth	5.97	5.87 E+33	-8.71 E+30	1.76 E+32
Mars	0.64	6.74 E+31	-3.75 E+30	6.13 E+31
Jupiter	1899.2	5.94 E+38	-3.22 E+29	2.85 E+30
Saturn	568.65	5.32 E+37	-9.55 E+28	6.24 E+29
Uranus	86.849	1.24 E+36	-2.38 E+28	1.10 E+29
Neptune	102.35	1.72 E+36	-9.61 E+27	3.54 E+28
Pluto	0.01305	2.80 E+28	-5.58 E+27	1.79 E+28

5 Spin angular momentum of planets

We propose here a formula for spin angular momentum of planets. This is given by

$$S = \frac{Gm^2}{\alpha_g c},$$

(35)

where $\alpha_g = 1.35\times 10^{-3}$ is the gravitational fine structure constant. The spin can be alternatively obtained from the relation

$$S = I\omega,$$

(36)

ω is the rotational angular frequency and I is the moment inertia. Since, I can't be readily calculated, our model could be the viable model to determine the planets spin and moment of inertia. Table 2 shows the spi

n of planets in our solar system. This can be compared with planetary spins that will be obtained from the future space exploration.

6 Electromagnetism and gravitomagnetism analogy

We have employed in this paper the analogy existing between electromagnetism and gravitomagnetism to calculate gravitomagnetic phenomena using their electromagnetic analogues. The results obtained can be compared with the gravitational calculations. This analogy is manifested mathematically in replacing GM^2 by kq^2 and their quantum analogues by replacing GM^2/c or kq^2/c by $\alpha_c\hbar_c$, where q, $\hbar_c$ and α_c are the characteristic charge, Planck and fine structure constants of the system under study (Arbab 2010a). While the electric component of a phenomena represents the main contribution, the magnetic component represents its correction, in terms of the coupling constant (G, $\hbar$) or the speed, v. One can now write the potential energy associated with the attractive force in Eq.(33) as

$$V_g = \frac{2\alpha_c^2\hbar_c^2}{M}\frac{1}{r^2},$$

(37)

Notice that the second term is proportional to the square of the Newton's potential. The mass M is the nuclues mass. Hence, we may call this a *quantum nucleus effect*. It arises in macroscopic (cosmic) as well as microscopic (atomic) systems. We can attribute this potential to the quantum effect. It is thus a quantum correction (or fluctuations). It is proportional to the relativistic effect. Consequently one can write the potential of the planets as a Taylor series as

$$V = -\frac{GmM}{r} + \frac{2\alpha_c^2}{Mr^2}\hbar_c^2 + O(\hbar_c^4),$$

(38)

This term is proportional to the term arising from the effective potential of an orbiting object about the central body. viz., $\frac{L^2}{2mr^2}$. The corresponding quantum potential will be $\frac{\hbar^2\ell(\ell+1)}{2mr^2}$. This can be compared with the spin orbit interaction in atomic physics, with $\hbar_c = \hbar, \alpha_c = \alpha$, where the second term is inversly proportional to r^3 (Griffiths 2004). This causes a shift in the electron energy levels. The contribution of the second term in Eq.(38) in hydrogen atom, with GMm replaced by kZe^2, will be

$$<\frac{1}{r^2}> = \frac{1}{(\ell + \frac{1}{2})n^3 a_0^2},$$

(39)

where a_0 is the Bohr's radius and ℓ is the orbital quantm number. Thus, the ground state contribution will be

$$E_0 = <\frac{2\alpha^2}{Mr^2}>\hbar^2 = \frac{4\alpha^2\hbar^2}{Ma_0^2}.$$

(40)

This energy may define the background energy acquired by any body in the vicinty of a central mass at a distance r. Hence, gravitational quantum fluctuations accompanied any centeal mass M. It is a fluctatution in the gravitational field surrounded any central mass. It could also be considered to arise from the gravitational radiation pressure. This effect is generally very small but measurable. At the atomic level this effect can be compared with other atomic effects present.

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